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SERIE AZUL

State-of-the-Art in Low-Resolution Face Recognition Methods

Mairelys Hernández-Durán, Yenisel Plasencia-Calaña, and Heydi-Méndez-Vázquez

RT_075 **mayo 2015**

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State-of-the-Art in Low-Resolution Face Recognition Methods

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Abstract. Face recognition has been studied for years due to its wide range of applications. Under controlled environments with cooperative subjects the recognition accuracy is satisfactory. However, there is a growing interest in real applications such as surveillance and video protection in which face regions tend to be small because subjects are far from cameras. In such scenarios, the recognition accuracy of face recognition is still unsatisfactory because of low resolution (LR) image quality. Low-resolution face recognition (LR FR) have the purpose of recognize faces from small size or poor quality images with varying pose, illumination, expression and others. This report provides a survey of different methods to recognize faces using LR face images and discusses some issues related to this topic.
Keywords: low-resolution, face recognition, super-resolution, face hallucination, feature extraction.

Resumen. El reconocimiento de rostros ha sido estudiado durante años debido a su amplia gama de aplicaciones. Bajo entornos controlados y con la cooperación de los sujetos las tasas de reconocimiento son satisfactorias. Sin embargo, existe un creciente interés en aplicaciones reales como vigilancia y video-protección en las que el tamaño del rostro tiende a ser pequeño ya que los sujetos se encuentran alejados de las cámaras. En estos escenarios, la precisión del reconocimiento facial sigue siendo insatisfactoria debido a la calidad de las imágenes de baja resolución (BR). El reconocimiento de rostros a partir de imágenes de baja resolución (RR BR) tiene el propósito de identificar imágenes de rostro de pequeño tamaño o poca calidad que presentan variaciones de pose, iluminación, expresión y otras. Este reporte ofrece un análisis de diferentes métodos para reconocer rostros utilizando imágenes de BR y discute algunas cuestiones relacionadas con este tema.
Palabras clave: baja resolución, reconocimiento de rostros, superresolución, alucinación de rostros, extracción de rasgos.

1 Introduction

Certainly, the biometrics applications for face recognition have achieved high recognition accuracy under controlled environments with frontal images. Nevertheless, in real applications face image size tend to be small and in such cases, the image do not have good definition of facial features. Discriminatory features present in the facial images used for distinguishing one person from another are lost due to the decrease in resolution resulting in unsatisfactory performance. As a result, these images with LR affect the performance of traditional face recognition systems. It was shown in [1] that traditional methods do not perform well when face images have relatively LR due to the fact that they are based on high resolution(HR) face images.

LR FR aims at recognizing face images with LR and variations such as pose and illumination. This is a more difficult task due to the fact that the degradation in resolution leads to the lack of effective features produces more noise to the image and cause dimensional mismatch when having to deal with different resolutions. To deal with the last one, three general approaches can be considered: interpolation, down-scaling and unified feature space. The first approach can be feasible under high-resolution, but may drop in performance confronting with much lower resolution because it does not introduce any new information in the process. In the second approach it reduces the amount of available information, especially the high-frequency information mainly for recognition. Finally, the third approach seems to be direct for solving the mismatch problem but it is difficult to find the optimal inter-resolution space.

Resolution indicates the information amount in pixels in an image and allows establishing a detail level or quality of the image. Although several criteria have been used in the literature to determine when a face image is from low-resolution, in the area of face recognition the majority established a low-resolution face image by the size of the face into the image; so the distance of subjects from the camera is an important measure.

With the aim to recognize faces from LR face images, several methods have been developed; being the super-resolution (SR) one of the more used approaches. This technique is usually employed to recover the lost information in the source image. SR methods produce a reconstructed high-resolution image from a low-resolution one or a sequence of low-resolution images by making assumptions about the image structure or content. It requires the recovering of lost high-frequency information occurring during the image formation process. Since 2005, for including facial features into an SR method as prior information, many researchers have studied Simultaneous Super-Resolution and Recognition (S2R2) [2]. At present, resolution-robust feature representation methods have been considered. However, all of them are limited to different constraints and do not completely solve the problem of recognizing subjects using low-resolution face images. Recently, Zou and Yuen [3] proposed the very low recognition problem (VLR), where the resolution of the face images to be recognized is lower than 16x12 pixels.

In order to have a clear idea on various LR FR methods, it is important to evaluate them based on certain evaluation criteria with some standard LR face databases. Unfortunately, this is seldom achieved due to the lack of general criteria and databases originally developed for LR FR. At present, this kind of methods is just evaluated based on high resolution face recognition criteria and databases. Therefore, a few standard LR face databases must be necessarily built for fair comparisons in future works.

2 Challenges in Low-Resolution Face Recognition

Face recognition is a difficult task, even more because it has to lead with face variations such as pose and expression. Many methods have been proposed for reducing the effects of the changes, such as the Discrete Cosine Transform (DCT) [4] on an edge map. However, most of the successful techniques could not be efficiently applied to LR data. Then, low resolution face images, as a new sub-area in face recognition, presents a greater challenge to recognition process.

Fig. 1. Architecture of a face recognition system.

Although many efforts have been made to improve LR FR methods, some specific problems still exist in real applications, especially in surveillance scenarios. The figure 1 shows the standard face recognition system including preprocessing, facial representation, feature extraction, and feature classification. All these stages have intrinsic problems which affects the performance of methods.

Preprocessing

Thus, alignment is one of the most important preprocessing issues in face recognition, especially in LR FR. Manual alignment is very difficult in applications such as video surveillance and the state-of-the-art face detectors such as AdaBoost [5] remain poor in detecting LR face images. To cope with this problem, some researchers attempted to propose automatic alignment techniques [6].

Facial representation

Most of the existing LR FR methods assumed the constrained cases such as frontal pose, good illumination, and neutral expression, while few methods focused on facial representation against different variations. For example, Chang et al. [7] examined the effect of resolution reduction with illumination variations. The majority of the methods only considered single variation by preprocessing or selecting the most suitable samples before recognition, that is to say they do not cover multiple variations.

Feature extraction

Most of the effective features used in HR FR such as texture and color may fail in LR case. Thus, it is difficult to find resolution-robust features for LR FR, especially under facial and environmental variations. Lei et al. [8] proposed a novel texture descriptor named Local Frequency Descriptor (LFD) based on Local Binary Patterns (LBP) and Fourier transformation.

Feature classification

Noise affection, misalignment, and lack of effective features commonly exist in face recognition system no matter HR or LR case. However, a dimensional mismatch problem in the traditional classification framework is the most essential problem in LR FR. Li et al. [9] proposed the coupled locality preserving mappings (CMs/CLPMs) model to build a unified feature space including SR and robust features for increasing the discriminability and generalization.

Table 1. Comparison between low-resolution and high resolution methods.

Criteria	**Low-resolution face recognition methods**	**High-resolution face recognition methods**
Advantages	Lower computational cost and storing	More features, more information, and less noises
Disadvantages	More noises, less information, and fewer available tools	Higher computational cost, and storing
Principal methods	MDS[14], S2R2[2], LFD[7], CLPMs[9]	LDA [71], PCA [28], LPP[38]

There are two paradigms for face recognition of low resolution faces. One is to use SR algorithms to enhance the image before recognition. On the other hand, it is possible to match in the low resolution domain by down-sampling the training set, but this is undesirable due to features important for recognition depend on high frequency details that are erased by down-sampling. In [2] they showed that the approach of matching in the low-resolution domain is better than applying SR when faces are of very low resolution.

3 Classification

Although some methods may overlap category boundaries, low-resolution face images methods can be divided into two main groups: SR methods and resolution-robust feature methods (see figure 2).

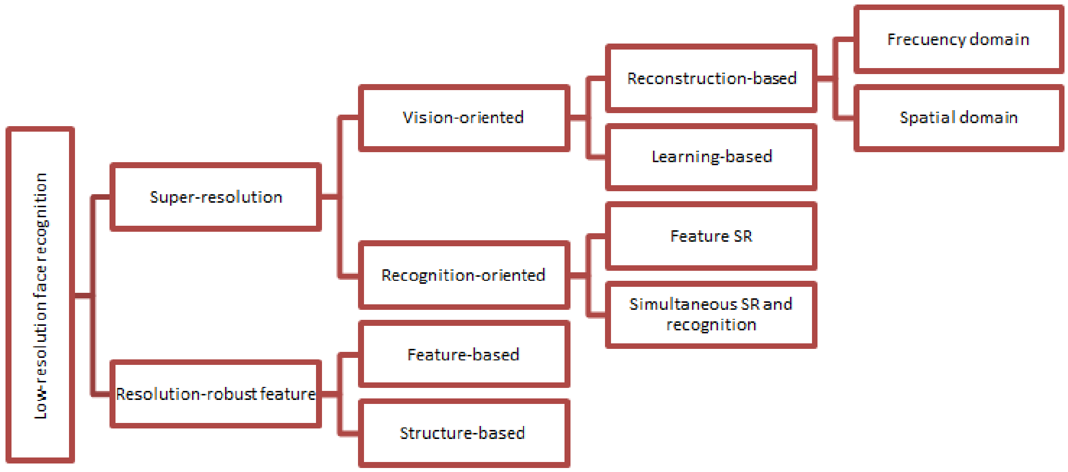

Fig. 2. Categorization of low-resolution face recognition method.

SR is used at first to synthesize the higher-resolution images from the LR ones, and then traditional HR methods could be used for recognition. This approach is considered from two criteria: vision-oriented methods (with the aim to improve visual quality) and recognition-oriented (from a recognition discriminability perspective). However, most of these methods [10] tried to enhance face appearance but failed to optimize face images from recognition perspective. Recently, a few attempts were made to achieve these two criteria under very LR case [3]. Vision-oriented methods can also be divided into two groups: reconstruction-based methods and learning-based methods. The first one can be addressed by frequency domain or by spatial domain. Recognition-oriented can be considered from two points of view further: feature SR [11] and S2R2 [2].

Resolution-robust feature methods consist on extracting the discriminative information from LR images. These methods can be divided into two groups further. One is feature-based method in which the resolution-robust features, such as texture [12], and subspace [13] information, are used to represent faces. However, some features used in traditional HR methods are sensitive to resolution. The other is structure-based method, e.g., multidimensional scaling (MDS) [14] in which the relationships between LR and HR are explored in resolution mismatch problem.

4 Super-Resolution

Face SR is a technique to reconstructs high-resolution face images from low-resolution face images, with the help of a-priori information of a sample dataset. It can restore the detail information of face features and enhance the resolution of poor quality face images, so it has an important role in improving the perceptual quality of face images.

SR has gained much more attentions in comparison to the development of resolution-robust face recognition methods, due to many problems that degrade the quality of face images in LR case. The problem here is that, as resolution decreases, SR becomes more vulnerable to environmental variations and it introduces distortions that affect recognition.

There are some ways of performing SR algorithms, but most of them are variations of two main approaches. Several approaches use single-frame SR which uses prior training data to enforce SR over a single low-resolution input image. Few methods deal with the multi-frame SR in which the HR image is derived from several LR observations of the scene which are typically aligned with sub-pixel accuracy. This method consist of two main stages, firstly estimating motion parameters between two images referred as registration, and secondly projecting the low-resolution image into the high resolution grid referred as reconstruction.

Hadid et al. [15] found that hidden Markov model (HMM) based methods with long sequences performed better than with short ones in both LR and HR, which means abundant information for recognition is included in video. Demirel et al. [16] used SR on different wavelet subbands of localized moving regions and composing the super-resolved subbands using inverse DWT to generate the respective enhanced high resolution frame. Dedeoglu et al. [17] proposed the concept of video hallucination by exploiting spatio-temporal regularities. Wheeler et al. [18] adopted a sequence of video frames represented by the active appearance model (AAM) for LR FR. Video-based feature extraction or video-to-video matching will provide a promising way for addressing the LR problem. Furthermore, multimodal biometric recognition systems, including LR FR may be used for recognition at a distance in the future.

In the last decade, most of the conventional SR methods, called vision-oriented, applied reconstruction followed by recognition. Theoretically, applying SR technique on the low-resolution face image, the reconstructed high-resolution image can be used for face recognition. However, this idea works well only if the input face image is frontal and captured under good illumination. Most of these techniques attempt to obtain a good HR reconstruction and are not optimized with respect to recognition performance. Recently, some researchers focused on SR mainly for recognition, called as recognition-oriented SR obtaining promising results for LR classification.

The landmark works in this category are face hallucination [19] and S2R2 [2]. The former proposed an algorithm to learn a prior on the spatial distribution of the image gradients for frontal images of faces. The gradient prior is learned using a collection of high resolution training face images. The latter is based on extracting a high-resolution template that simultaneously fits SR as well as the face-feature constraints.

4.1 Vision-Oriented Methods

This kind of methods produces a reconstructed high-resolution image from one or several LR images assuming information about the structure of the image or their contents [19]. Most of the vision-oriented SR methods have attempted to minimize mean-squared error (MSE) or maximize signal to-noise ratio (SNR) between the original HR and the reconstructed SR images. However, in face recognition, these SR approaches may not perform well, as most face recognition systems rely on the ability to identify key facial features, typically captured by the high-frequency content. Obtaining a higher SNR does not necessarily contribute to a higher recognition rate since high fidelity reconstruction of low-frequency content may dominate the image [20].

Thus, the goal of vision-oriented SR methods is to obtain a good visual reconstruction, but not usually designed from recognition perspective. Despite their improvements, this kind of methods has limitations such as inconsistent targets in restoration and recognition. Moreover, minimizing the reconstruction error in image restoration may not always guarantee good performance of the subsequent face recognition. A promising way to further improve the robust performance of SR for recognition is to embed SR into recognition.

4.1.1 Reconstruction-Based Methods

The conventional SR methods attempt to recover the source image by solving the ill-posed inverse problem, $y = Hx + v$, where x is the unknown HR image to be estimated, y is the observed LR image, H is the degradation matrix, and v is the additional noise vector. Under the scarcity of observed LR images, the inverse process is an undetermined problem, thus the solution is not unique. To find a solution, some prior information of the images is often incorporated into the reconstruction process.

Reconstruction-based methods [21] produce an HR image under the constraint that the smoothed and down-sampled version of the reconstructed HR image is close to the input LR image. For example, back-projection algorithm iteratively minimizes the reconstruction error. But, those algorithms rarely avoid artifacts along the strong edges. Hence, the basic idea in this approach is reconstructing HR images simulating the image formation process. Its performance is affected by the noise level present in the face image, the accuracy to estimate the Point Spread Function (PSF) and the alignment accuracy.

In the preprocessing step this approach only aims at minimizing the reconstruction error between restored images and the ground-truth, without any consideration of the subsequent recognition target. The restoration process with inconsistent target cannot always improve the performance of face recognition as much as we expected. Besides, the SR and deblurring algorithms used are usually computational complex, which is not ideal for some real-time systems.These groups of methods have several limitations when the magnification factor increases as referred Baker et al. [22] because some approaches do not incorporate any prior information about the super-resolved images. Recently, Nasrollahi et al. [23] tried to improve the magnification factor from about two to almost four by using multilayer perceptron. Moreover, the majority of these methods are more suitable for synthetizing local textures, so they are generally used to generic objects or scenes instead of face images.

Reconstruction-based methods can be addressed into two different approaches: frequency domain and spatial domain. The former is based on modifying the Fourier transform while the latter is based on direct manipulations on image pixels. Here, we briefly summarize the two ways.

Frequency Domain Methods

Tsai and Huang are the pioneers of SR [24] idea who used the frequency domain approach. Reconstruction of high frequency details is the goal of these methods.

Lucchese et al [25] presented a method for estimating planar roto-translations that operates in the frequency domain. They utilized separately rotational and translational components of the Fourier transform. A detailed inspection of frequency domain methods appears in [26]. Zhang et al. [27] performed SR in frequency domain with inferring DCT coefficients instead of estimating pixel intensities in spatial domain. Alternating component coefficients in DCT were inferred by the Markov network of low-level vision. Subspace methods are also applied to restrict the reconstructed HR image locating within face subspace, such as Principal Components Analysis (PCA) [28] and kernel PCA subspace [29].

These methods have been used in practice because they are relatively simple and have a low computational cost. Still, the majority of this kind of methods focused on frontal faces, and fails to deal with unconstrained variations. Moreover, this approach restricts the translational motion as the DFT (Discrete Fourier Transformed) assume uniformly spaced samples. Moreover, prior information is often difficult to express in the frequency domain, which is an essential element to achieve good results in SR.

Spatial Domain Methods

These methods handle directly the image pixels. Although some of these methods have shown to be successful for high-resolution images based on local features, such as Gabor wavelets [30] and LBP [31]; facial texture SR in the spatial domain could not significantly improve the performance of low-resolution image recognition methods [32]. This approach allows more flexibility to adopt all type of degradation model image in comparison with frequency domain methods.

Fuzzy registration is a commonly used technique for reconstruction in spatial domain, which exploits the correlation between pixels of the target image and other LR images. It performs image registration and reconstruction simultaneously with a weighted average of neighbors.

Interpolation method is another technique considered into this approach to increase resolution of images. This kind of methods allows calculating unknown numerical values through other values already known using specific algorithm. The basic idea is having a certain number of points acquired through sampling and from them, making a function that adjusts them. Both bilinear and bicubic interpolations are the most used methods in this group due to the simplicity and low computational cost.

However, with the increasing magnification factor, they are prone to generate overly smooth edges. Concluding, though being extremely fast, it suffers from excessive smoothing out of edges, and producing ringing artifacts during reconstruction.

4.1.2 Learning-Based Methods

This technique, also called face hallucination, was first proposed by Baker and Kanade [19]. It employs a training database consisting of pairs of high and LR images samples to output hallucinated high-resolution faces. These methods are based on either global models or the patch approach. They exploit the prior knowledge between the high and its corresponding LR examples through so called learning process. Their technique ignores local details to focus on global information, so the resulting images lack detailed features. In this approach, the reconstructed high-resolution image shows significant blocking artifacts as the patch-based methods divide the image into small blocks that are uniformly overlapped and also because hallucination is performed on each block.

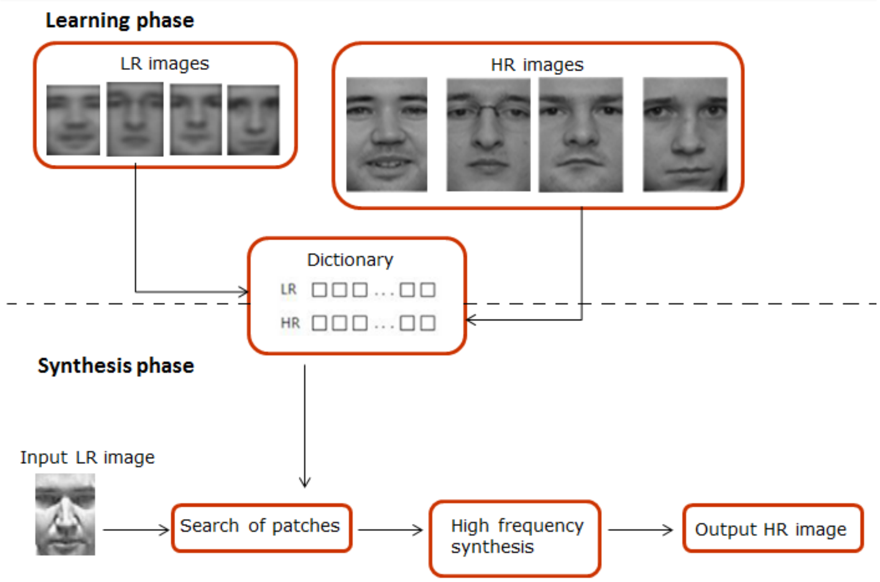

Fig. 3. Learning-based approach.

Figure 3 describes the basic concept of learning-based SR that is generally composed of two phases: learning phase and synthesis phase. At the learning phase, the training data, i.e., dictionary consisting of LR and HR patches is constructed. The LR and HR patch pairs are obtained from various training images. During the synthesis phase, the input LR image is super-resolved by using the dictionary. For each LR patch in the input image, its nearest neighbor LR patches are explored from the dictionary. The high frequency components of the input LR patch are synthesized using the best matched LR patches. [33]

Establishing a good learning model to obtain the prior knowledge is the key to the learning-based method. Another challenge in face hallucination is the difficulty of aligning faces at low-resolution.

Nowadays, the commonly used learning models include the PCA model [9], Markov model [31], and others. Wang and Tang [9] used PCA to express the input face image as a linear combination of low-resolution training face images, and the final high-resolution image is synthesized by replacing the low-resolution training face image with their high-resolution counterparts, making use of the same combination weights. Unfortunately, their technique ignores local details to focus on global information, so the resulting images are unclear and lack detailed features.

Usually, existing learning-based SR methods require hundreds of thousands of training examples for a reliable performance. However, such a dictionary size causes huge memory cost for storing the training samples as well as awfully large computational complexity in the matching process. Therefore, it makes conventional learning-based SR impractical in implementation and restrictive in applications.

In [34] they achieve fast image SR by reducing the size of trained dictionary. The reduced dictionary size makes it possible to significantly speed up SR processing and save the memory cost, while providing reasonable visual quality. They used Feature Match [35] and a common source image they showed a more accurate and much faster perform than other existing techniques. However, these results are for synthetic image SR, they don't have to lead with face variations and other related issues. Some researchers added constrains to the process to improve its performance [3, 36]. Although some methods may overlap category boundaries, Zou et al. [3] further categorized learning-based SR methods into two groups, namely example-based method [27, 29, 36, 37] and Maximum a Posteriori (MAP) based method [5, 9, 38, 39, 41].

Example-Based

Reconstructing a HR image from a linear or non-linear combination of training images is the goal of example-based methods. In this approach, it is important to determine the weight coefficients because they are useful to minimize linear or non-linear approximation error.

Most example-based SR algorithms usually employ a dictionary composed of a large number of HR patches and their corresponding LR patches. The input LR image is split into either overlapping or non-overlapping patches. Then, for each input LR patch, either one best-matched patch or a set of the best-matched LR patches are selected from the dictionary. The corresponding HR patches are used to reconstruct the output HR image. However, most of the existing algorithms are computationally intensive in finding the best match of LR–HR patch from a big dictionary. Furthermore, best-matched but incorrect patches will seriously degrade the reconstruction results.

Eigenface space [9], tensor space [37], and manifold learning techniques [38, 39] are used for example-based SR. Here, we briefly summarize them.

Wang et al. [9] proposed a representative example-based method and treated the hallucination problem as a transformation between LR and HR. They used PCA to fit an input LR face image as a linear combination of LR training images. The HR image was then synthesized by replacing the LR training images with their HR counterparts while retaining the same combination coefficients. However, the linear PCA model could not capture distinct structures of the input face efficiently and only focused on global estimation without paying attention to local details. Thus, the results lacked detailed features, and caused some distortions. Moreover, they designed a mask to avoid artifacts on hair and background, and performed hallucination in the interior region of the face. In fact, local modeling and appropriate smoothing can be adopted to handle these artifacts properly. Given that, the idea of two-step in [40] can be used to compensate high-frequency features for the work.

Compared with Wang's work [9] that worked on the eigenface space, Liu et al. [40] proposed a two-step approach in patch tensor space using a patch-based non-parametric Markov network locally to reconstruct high-frequency content. Assuming that the low-resolution space and high-resolution space share similar local distribution structure, the estimated parameters are used for synthesizing high-resolution images. To further enhance the quality of the HR image, the coupled PCA method was developed for residue compensation. While the method added more details to the face, it also introduced more artifacts. Therefore, whether to adopt residue compensation techniques and when to do them is critical for SR.

Manifold learning approaches suggest that the subspace of face images has an embedded manifold structure. The high-dimensional structure formed by HR face images is homeo-morphic with a geometric structure in LR space. It means that the features of LR and HR face images share a common topological structure, and thus, they are coherent through the structure. The basic idea is that small image patches in the low and high-resolution images form manifolds with similar local geometry in two distinct feature spaces. This method requires fewer training examples than learning based approach because the generation of a high-resolution image patch depends on multiple nearest neighbors. Some ideas of manifold learning such as locality preserving projection (LPP) [38] and local linear embedding (LLE) [39] are introduced into SR.

Zhuang et al. [41] developed locality preserving hallucination method based on LPP [42]. It combined LPP and Radial Basis Function (RBF) [43] together to hallucinate a global HR face. Compared with Wang's work [9], the hallucinated global HR face contained more detailed features. However, there were more noises in the local features such as contour, nostril, and eyebrow, because LPP resulted in the loss of non-feature information. To improve the details of the synthesized HR face, they developed a residue compensation method based on patch by neighbor embedding [39].Their approach does not depend on just one of the nearest neighbors in the training set. They represent each low or high resolution image as a set of small overlapping image patches.

In addition, some methods performed example-based SR on single-frame LR face image [5, 44]. Recently, Hu et al. [44] also developed a single-frame SR method, like Liu's work [40]. They used both global and local constraints for hallucination; the difference was that their global model was derived from the non-rigid warping of reference face examples and the learning of the pixel structure. The warping could capture a moderate range of face variations. And the effects of warping errors were reduced by the adaptive weighting in the local prior model. Thus, the method could infer more faithful individual structures of the target HR face.

Maximum a Posteriori (MAP)

In the last decade, this approach has achieved great popularity because models bring necessary soft-constrains to ensure an acceptable reconstruction under conditions of poor or bad quality of the input data, and also allows the effective reconstruction of borders. It allows one incorporating prior constrains in form of probability density, which is essential to find high quality solutions.

To estimate this probability, different algorithms have different solutions. Baker et al. [3] first proposed the idea of face hallucination and led the precedent of learning-based method. They estimated the function probability by using an image Gaussian pyramid under Bayesian formulation. The method obtained high-frequency components from a parent structure based on training face images; however, it intrinsically relied on a complicated statistical model.

Similar to Baker's work, Capel et al. [45] also used MAP estimators, with the difference that Capel divided a face image into six unrelated parts, and applied PCA on them separately. Liu et al. [36, 40] proposed a two-step statistical method. They applied PCA linear model and a Markov random field prior to maximize a probability for obtaining a local feature. It depends on an explicit down-sampling function, which is sometimes unavailable in practice.

Many methods treating face hallucination as a two-step problem have been proposed [46, 47]. They all perform the following two-step process. First, a global face image containing low-frequency information is obtained, which looks smooth and lacks some detailed features. Second, a residue face image keeping high frequency information is synthesized. Thus, the residue image is piled onto the global image to get the final super-resolved face images. For example, Li et al. [47] used a MAP criterion for reconstructing both the global image and the residual image. Jia et al. [46] proposed a unified tensor space representation for hallucinating low-frequency and middle frequency information, and then recovered high-frequency part by patch learning.

4.2 Recognition-Oriented Methods

The essence of the recognition-oriented SR methods is to satisfy the need of recognition with LR images. It embeds the elements of SR methods into face recognition. That is to say, fuses the models of the image formation process and the prior information, together with feature extraction and classification to design methods for recognition [2, 10, 47, 50].

Compared with vision-oriented SR, recognition-oriented SR maybe more suitable for LR FR because it simultaneously performs SR and feature extraction with the direct goal of recognition and later performs feature SR with the aim of reconstructing low-frequency and high-frequency content for recognition.

Some successes have been achieved by recognition-oriented SR methods such as S2R2 and feature SR. However, they just provide the framework for recognition oriented SR, and their recognition performances largely depend on different reconstruction regularization models and feature extraction techniques. Some common problems are still unsolved in these methods. For example, it is unclear what kind of reconstruction regularization method is more appropriate for recognition. In addition, feature extraction is known to be sensitive to large appearance changes due to pose, illumination, expression, etc. To combine SR and feature extraction is also an important issue for the future work. A possible way to handle these problems is to adopt more robust feature extraction techniques.

4.2.1 Feature Super-Resolution

It was proposed by Li et al. [47] to reconstruct HR features instead of HR images for face recognition. The kernel version of Support Vector Data Description (SVDD) [58] was used to synthesize HR discriminative features both for vision and recognition perspective [48]. A spherically shaped decision boundary around a set of objects is constructed by a set of support vectors describing the sphere boundary. It has the possibility of transforming the data to new feature spaces without much extra computational cost. However, the method is only applicable on frontal faces and its generalization ability remains doubtful.

Two representative methods in this group are Nonlinear Mappings on Coherent Features (NMCF) [50] and Discriminative Super-Resolution (DSR) [3]. Both of them introduce classification discriminability into SR process.

Huang et al. [50] proposed NMCF with Canonical Correlation Analysis (CCA) [51] to establish coherent features between LR and HR images represented by PCA. Here, the problem of SR of feature domain for face recognition is formulated as the inference of the HR domain feature C_h from an input LR image I^{L} given the training sets of HR and LR face images I^H and I^{L}, given by:

$$I^H = \{\mathrm{I}_i^{\mathrm{H}}\} = [\mathrm{I}_{1,}^{\mathrm{H}}\ \mathrm{I}_2^{\mathrm{H}}, \dots \mathrm{I}_m^{\mathrm{H}}], \tag{1}$$

$$\mathrm{I}^{\mathrm{L}} = \{\mathrm{I}_i^{\mathrm{L}}\} = [\mathrm{I}_1^{\mathrm{L}}, \mathrm{I}_2^{\mathrm{L}}, \dots, \mathrm{I}_m^{\mathrm{L}}], \tag{2}$$

where m denotes the size of the training sets. The first step consists on extract the feature vectors X^H and X^Lusing PCA corresponding to the training HR images and the training LR images respectively:

$$\mathrm{X}^{\mathrm{H}} = \{\mathrm{x}_i^{\mathrm{H}}\}_{I=1}^{m} \in \mathrm{R}^{p\ x\ m}, \tag{3}$$

$$\mathrm{X}^{\mathrm{L}} = \{\mathrm{x}_i^{\mathrm{L}}\}_{I=1}^{m} \in \mathrm{R}^{q\ x\ m}. \tag{4}$$

Specifically, from the PCA feature training sets $\mathrm{X}^{\mathrm{H}}$and X^{L} they first subtracted their mean values $\overline{\mathrm{X}}^{\mathrm{H}}$ and $\overline{\mathrm{X}}^{\mathrm{L}}$ taken,

$$\widehat{\mathrm{X}}^{\mathrm{H}} = [\widehat{\mathrm{X}}_1^{\mathrm{H}},\ \widehat{\mathrm{X}}_2^{\mathrm{H}}, \dots \widehat{\mathrm{X}}_m^{\mathrm{H}}], \tag{5}$$

$$\widehat{\mathrm{X}}^{\mathrm{L}} = [\widehat{\mathrm{X}}_1^{\mathrm{L}},\ \widehat{\mathrm{X}}_2^{\mathrm{L}}, \dots \widehat{\mathrm{X}}_m^{\mathrm{L}}]. \tag{6}$$

CCA finds two base vectors VH and VL for datasets $\hat{X}^{H}$ and $\hat{X}^{L}$ in order to maximize the correlation coefficient between the coherent vectors $C^{H} = (V^{H})^{T}\hat{X}^{H}$ and $C^{L} = (V^{L})^{T}\hat{X}^{L}$.

To find the base vectors VH and VL, they defined four main matrices: $C_{11} = \hat{X}^{H}\hat{X}^{H^{T}}$, $C_{12} = \hat{X}^{H}\hat{X}^{L^{T}}$, $C_{22} = \hat{X}^{L}\hat{X}^{L^{T}}$, and $C_{21} = \hat{X}^{L}\hat{X}^{H^{T}}$ as their between-set covariance matrices. Then, computing:

$$R_1 = C_{11}^{-1} C_{12} C_{22}^{-1} C_{21}, \tag{7}$$

$$R_2 = C_{22}^{-1} C_{21} C_{11}^{-1} C_{12}. \tag{8}$$

V^{H} is made up of the eigenvectors of R_1 when the eigenvalues of R_1 are ordered in descending order. Similarly, the eigenvectors of R_2 compose V^{L}. We obtain the corresponding projected coefficient sets:

$$C^{H} = \{c_i^{H}\}_{I=1}^{m} \in R^{p\,x\,m}, \tag{9}$$

$$C^{L} = \{c_i^{L}\}_{I=1}^{m} \in R^{q\,x\,m}, \tag{10}$$

of the PCA feature sets X^{H} and X^{L} projected into the coherent subspaces using the following base vectors:

$$c_i^{H} = (V^{H})^{T}\hat{X}_i^{H}, \tag{11}$$

$$c_i^{L} = (V^{L})^{T}\hat{X}_i^{L}. \tag{12}$$

The correlation between the two sets C^{H} and C^{L} is increased after the transformation and the relationship between HR and LR features are more exactly established in the coherent subspace. Motivated by Zhuang's work [41], they also applied RBF mapping to build the regression model by adopting the advantages of RBF, such as fast learning and generalization ability. This method was evaluated on 12×12 with FERET [52] database and obtained the accuracy of 84.4 % compared with 36.9 % of the PCA baseline method. In regression based methods the idea is to learn a mapping from input LR images to target HR images, for example image pairs using kernel ridge regression.

Recently, Zou et al. [3] proposed recognition of low resolution face image method with nonlinear variations. It learns the nonlinear relationship between LR face image and HR face image in nonlinear kernel feature space. They proposed the DSR method with new data constraint and discriminative constraint. It modeled the SR problem as a regression problem under very LR case such as 16 ×12 and 7 × 6.

In this work the SR problem is modeled by learning the relationship R from the given training data and the reconstructed HR image is recovered by applying R on the testing image. Instead of recovering the HR image directly, they learned the relationship R between HR image space and very LR image space.

In order to find a better subspace induced by R with other additional constraint(s), so that the reconstructed HR images have more discriminative features, discriminative constraint is designed to the relationship learning based SR algorithm as follows:

$$\dot{R} = \arg min_{R'} \frac{1}{N}\sum_{i=1}^{N}\left\|I_h^{i} - R' I_l^{i}\right\|^{2} + \gamma d(R'), \tag{13}$$

where γ is a constant to balance the above two terms. The HR image can be reconstructed after $\dot{R}$ is determined. The HR images reconstructed by $\dot{R}$ locates in a subspace where they can be better linear separable. Therefore, the HR image reconstructed by $\dot{R}$ will contain more discriminability and be better for recognition purpose.

DSR shows its superiority from both visual quality and recognition performance. For example, SR results on 16×12 with Extended Yale B database are shown in Fig. 7. Also, DSR obtained the recognition accuracy of 73.5 % on $7{\times}6$ with CMU PIE [53] database in comparison with 40.5 % in the PCA baseline method.

Zhifei et al. [20] established a brief comparison between DSR and NMCF. Both DSR and NMCF require a training set containing LR and HR image pairs to learn the nonlinear mappings from LR to HR feature space, followed by the reconstruction of SR images or features. Compared with NMCF, DSR performs more efficiently when LR images are used for training/gallery sets. Conversely, when HR training/gallery sets are used NMCF outperforms DSR.

4.2.2 Simultaneous super-resolution and recognition methods

Multimodal tensor super-resolution (M2TSR) [54] and S2R2 [2] are two representatives methods in this group.

Multi-linear analysis [55] is a general extension of the traditional linear methods such as PCA. Instead of modeling the relations within vectors or matrices, multi-linear analysis provides a means to investigate the mappings between multiple factor spaces.

Tensor Face [55] has been proposed for a multi-linear analysis to model explicitly the multiple modes of variations in facial images and their inter-relationships. Although experiments suggested improved recognition performance over traditional approach, the recognition rates based on tensor face methods decrease dramatically with low-resolution inputs.

Jia et al. [54] performed multimodal face image SR for recognition in tensor space. They integrated the task of SR and recognition by directly computing a maximum likelihood identity parameter vector in high-resolution tensor space for recognition. Although it did not simultaneously cope with pose and illumination variation, face hallucination and recognition were unified in this way. However, its disadvantage was that the tensor manipulations for reconstruction demanded high computation expenses. This method is a tensor extension of the SR problem for the single modal LR face image. The consideration of multimodality could contribute to LR FR.

More recently, some researchers propose a new algorithm which could simultaneously carry out SR and feature extraction for LR face recognition [2]. They proposed S2R2 method with the main purpose of obtaining a suboptimal output HR image from both reconstruction and recognition perspectives. In this approach, face features are included in an SR method as prior information. S2R2 consist on a two-step approach that uses constraints of an SR algorithm and features from a classifier trained with images having the desired resolution. They showed that the performances of conventional SR methods were degraded under very LR case.

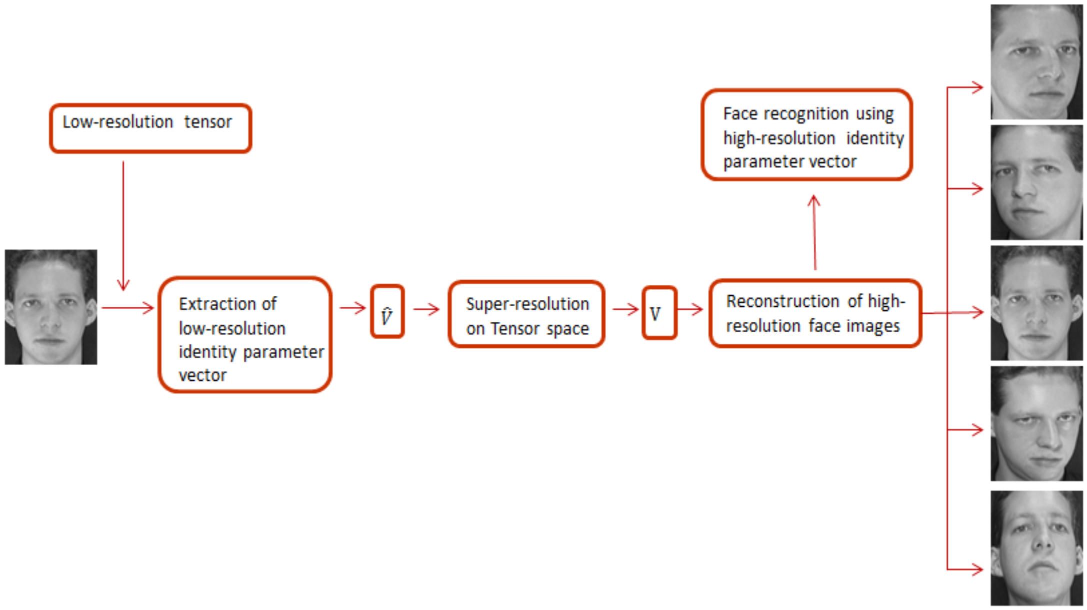

Fig. 4. Multi-modal super-resolution and recognition process in tensor space using a multi-view super-resolution example.

Formula (14) denotes the base model of S2R2, y_p, $f_g^{(k)}$ and x denote the input LR probe image, the gallery image in the kth class, and the output HR image, respectively; B, L, and F represent operators for down-sampling, smoothness and feature extraction, respectively; besides, α and β are the regularization parameters. The goal of the S2R2 model is to obtain a suboptimal output HR image x for satisfying the need of vision and recognition simultaneously.

$$\|B_x - y_p\|^2 + \alpha^2\|L_x\|^2 + \beta^2 \left\|F_x - f_g^{(k)}\right\|^2. \quad (14)$$

In addition, the base S2R2 model was improved by involving the cases of multi-frames or multi-cameras version $B^{(i)}$. Furthermore, the base SR prior model $l^{(k)}$and feature extraction F_L were modified based on multi-resolutions version (l or L). The modified model is shown in (7), where α, β, γ are the regularization parameters, and B denotes the image formation process. The base S2R2 model and the improved version tested on CMU Multi-PIE [56] database on 6×6 obtained the accuracies of 62.8 % and 73 %, in comparison with the PCA baseline method at 47.1 %.

$$\left\|B^i x - y_p^{(i)}\right\|^2 + \alpha^2\left\|L_x - l^{(k)}\right\|^2 + \beta^2 \left\|F_x - f_g^{(k)}\right\|^2 + \gamma^2 \left\|F_L B_x - f_L^{(K)}\right\|^2. \quad (15)$$

Compared with the general indirect SR methods, S2R2 improves identification accuracy and gets promising results on 6 × 6. However, the parametric optimization needs to be repeated for each gallery image in the database, especially for large databases; thus, their formulation is quite time-consuming. Also, this method assumes that gallery and probe images are in the same pose, frontal or localized perfectly, directly resulting in its inefficiency under many general scenarios. Therefore, how to obtain the appropriate regularization parameters and reduce the computational complexity are two important issues in this model.

5 Resolution-Robust Feature Methods

The difficulties of finding the effective features in LR case make face recognition more complicated. Some typical features in HR case such as texture, shape, and color may fail in the LR case. Still, exploring the potential of these features for LR FR or building inter-resolution space may provide a promising direction for solving this problem.

Resolution-robust feature representation methods are classified into two groups: feature-based method [7, 13, 57] and structure-based method [8, 14, 58]. Compared with feature-based methods, the structure-based approach is more suitable for offline training, but they are mainly used for a single resolution application with the balance between efficiency and speed. The landmark works are color feature (Choi et al. [57]) and CLPMs.

Although many researchers addressed resolution-robust feature representation, it is still a difficult task due to different variations. Most methods included into this group deal only with one variation and not with multiple variations. Compared with local features, global features are more sensitive to illumination variation, which is a significant obstacle for LR FR application. With regards to pose and expression variation, local features are more susceptive than global features. A possible way to further improve the robustness may lie in the combination of local based and global-based features. However, what features should be combined and how to combine them for concentrating their advantages are the future issues for LR FR.

5.1 Feature-Based Methods

This method identifies an LR face directly using the features extracted from probe images in resized forms. However, all the existing resolution-robust features are improved from the features used in HR FR, such as the improved color space [57] and the improved local binary pattern descriptor [7]. The feature-based methods can be used for multiple resolutions from HR to LR, but they need online training. We further can divide feature-based methods into two categories, that is, the global feature-based method and local feature-based method.

5.1.1 Global Feature Based Methods

In this method, the whole LR probe image, represented by a single high-dimensional vector containing the global low frequency information, is taken as input. The advantage of this method is to implicitly preserve all the detailed texture and shape information, which is useful for recognizing LR faces. On the other hand, this method is also easily affected by variations such as pose and illumination similar to HR FR.

Since many face recognition systems use an initial dimensionality reduction method, Gunturk et al. [28] proposed eigenface-domain SR in the lower dimensional face space. Moreover, some other approaches such as SVDD [59] have been used to address this issue. Although PCA is commonly used for facial images representation in global face SR, the features extracted by PCA are holistic and difficult to have semantic interpretation. Besides PCA is not a good factorization method for synthesis and reconstruction. To cope with this, Chengdong et al [60] introduce non-negative matrix factorization (NMF) to extract face features and propose a global face super-resolution with contour region constraints (CRNMF) for improving the quality of SR facial image. This method resolves many issues of the traditional method based on PCA. It reduces dependence on the pixels and preserves face structure similarity given that the contours of the human face contain the structural information.

Color features are the representative global features. Choi et al. [57] first demonstrated that color-based features could significantly improve LR FR recognition performance compared with gray-based features. The idea was based on the boosting effects of color features on low-level vision [61]. A new metric called variation ratio gain (VRG) was further defined to prove the significance of color effect on

LR face images within the subspace face recognition framework. They found that color components can compensate a decreased extra-personal variation by intensity component with LR, so RQCr color space was selected for LR FR. However, there is no formal demonstration that RQCr is more efficient for LR case in comparison with other color spaces.

Therefore, to efficiently use color-based features for boosting intensity-based features is still an open issue. It is known that reducing the correlation of different color components is certainly helpful to HR FR, and even LR FR. Yang et al. [62] investigated the potential efficiency of color spaces, and proposed various normalized spaces such as the improved YRB space to enhance face recognition.

Choi et al. [63] improved their work and proposed a color feature selection method by boosting-learning framework. Thirty-six different color components were used to form a color-component pool, and a weighted fusion scheme was used to fuse the selected color features at the feature level. The method was successfully evaluated on very LR images with SCface [64] database. It improved the accuracy with RQCr space from 49.61 % to 62.78 % with the new color pool. The experiment indicated that the framework of color fusion was perhaps beneficial to LR FR. However, the role of color features for LR images is degraded by serious illumination variations despite of their successes in face recognition.

Abiantun et al. [13] adopted the kernel class-dependence feature analysis (KCFA) method [65] for dealing with very LR case. KCFA used a set of minimum average correlation energy filters to exploit higher order correlations between training samples in the kernel space, and obtained the accuracy of 27.1 % on 8×8 images compared with the PCA baseline method of 12 % on HR images using FRGC [66] database Experiment 4.

5.1.2 Local Feature Based Methods

Compared with global method, local feature based method provides additional flexibility to recognize a face based on its parts, and is more robust to variations. In this approach the LR probe image is represented by a set of low-dimensional vectors containing the local high frequency information.

Ahonen et al. [58] adopted local phase quantization (LPQ) method based on the assumption of the PSF. The method used the phase information of Fourier transformed images for LR FR, revealing that LPQ information in the high-frequency domain was almost invariant to blurring. Afterwards, Lei et al. [7] made an improvement on LPQ and proposed LFD using not only phase information, but also magnitude information. Furthermore, the relative relationships between phase information were adopted without the assumption of the PSF instead of the absolute value. Also, a uniform pattern mechanism [31] was introduced to improve the performance.

5.2 Structure-Based Methods

This kind of methods focuses on constructing the relationships between LR and HR feature space for facilitating direct comparison of LR probe images with HR gallery ones from a classification perspective. The method aims to build the holistic framework for LR matching by solving the dimensional mismatch. A coupled mapping (CM) [8] is one kind of structure-based methods. Obtaining an inter-resolution space or unified feature space is the key for solving the mismatch problem in this approach.

5.2.1 Coupled Mappings

Choi et al. [58] first pointed out the dimensional mismatch problem, and proposed eigenspace estimation (EE) techniques for obtaining a common LR feature space for matching between LR and HR. Then Li et al. [8, 67] proposed a more general framework called unified feature space based on CM, as follows:

$$J_{(A_L - A_H)} = \sum_{i=1}^{N_t} \left\| A_L^T l_i - A_H^T h_i \right\|^2. \quad (16)$$

In this model, l_i and h_i, represent an LR face image and an HR one, respectively; and A_L *and* A_H are two coupled mapping matrices. For LR FR, the mapping between each LR image and the corresponding HR image is expected to be as close as possible in the new unified feature space. In [67] they formulated the coupled metric learning as an optimization problem. They decided two transformations: the former maps degraded images to a new subspace, where higher recognition performance can be achieved; the other one maps normal images and class labels together to the same subspace for better class-wise feature representation. The coupled transformations are determined by solving an optimization problem.

Although CMs provides a promising framework for learning the relationships between LR and HR; it has a shortcoming in poor discriminability for classification. Therefore, Li et al. introduced the locality preserving objective [42] into nonparametric coupled mapping model, and proposed coupled locality preserving mappings (CLPM) method. It significantly improved the performance by involving the weight relationships among data points. However, CLPMs is sensitivity to the parameters and pose variations.

Other works with the aim of improving CMs have been developed. Ren et al. [68] adopted CCA with local discrimination criterion [69] to compute the two coupled mapping matrices. With the process of regularization on feature space, the method showed its superiority compared with CMs/CLPMs in both recognition accuracy and time complexity.

Recently, Ren at al. [70] further introduced the kernel tricks into CLPMs and proposed coupled kernel embedding (CKE) method for dealing with LR FR. By the kernel tricks, on one hand, CKE improved the classification performance; on the other hand, it increased the time complexity.

Similar to the work in [8], Biswas et al. [14] used MDS during training phase to embed LR images into a new Euclidean space in order to achieve the best distances between their HR counterparts. MDS is a technique used to extract a set of independent variables from a proximity matrix or matrices. MDS can be described as a set of techniques for interpreting similarity or dissimilarity data. In [14] they highlighted the pose problem involved in LR recognition. This is an important contribution for researches on LR FR. They evaluated MDS on CMU Multi-PIE (8 × 6) and SCface database (12 × 10), and obtained the accuracies of 52 % and 71 %, respectively. In their experiments for LR FR, MDS performed better than sparse representation based SR [71].

As a conclusion, we can say that CMs needs to improve its ability in discriminability and generalize CMs from single LR to multiple LR applications even for across all resolutions. Moreover, this approach needs to solve the eigenvalue decomposition problem and obtain the two optimal mapping matrices.

5.2.2 Resolution Estimation

Resolution estimation makes efforts for dealing with the dimensional mismatch problem. It determines the kinds of structures chosen for building a LR FR system. Wong et al. [72] proposed two innovations for the LR problem. One was the concept of an underlying resolution, which did not rely on the size of face image. The other was that the local features sensitive to resolution were exploited for LR classification. Based on the innovations, they proposed a resolution detection and compensation framework for dynamically choosing the appropriate face recognition system.

A similar method was proposed by Pedro et al. [73]. They developed the concept of estimating the acquisition distance in three different scenarios and the distance was taken as the weight to fuse two systems (PCA-SVM system and DCT-GMM system) at the score level. They demonstrated that training with medium distance images was a good way to control the performance degradation due to the varying distance.

5.2.3 Sparse Representation

Is first proposed by Wright et al. [71] and recently has become one of the standard methods of face recognition followed by many researchers for representing LR probe images using HR training images from a structure perspective.

This approach may provide a new theoretical framework for dealing with LR FR problem. It approach use an over-complete dictionary pair of low-resolution and high resolution patches. Initially, a pair of low-resolution and high-resolution dictionaries is co-trained. To perform the SR of a given LR image, each part of the image is compared to the LR dictionary and using the sparse coefficients on the LR dictionary, the HR patch corresponding to these LR patches satisfying certain spatial properties are combined to form the output.

Researchers cast the recognition problem as one of classifying among multiple linear regression models. If the number of features was sufficiently large, and the sparse representation was correctly computed, they demonstrated that the choice of features was no longer critical. It was right even in the down-sampled images, though their work was not specialized for the LR case. However, like most of the other methods, sparse representation also requires a large amount of training covering different variations. Aimed at this problem, the non-local prior may be employed to enrich the textural information [74]. They used the self-similarity prior of the testing image to generate virtual observed LR examples. A few studies start to gradually upscale the LR image since self-similarities works better on small up scaling factors [75]. However, they lack a more explicit layer-wise model. In [76] they developed a new layer-wise model, referred to deep network cascade, to upscale the input LR image layer by layer each layer with a refined SR result.

Recent progress has focused on the effectiveness of the l^1 norm for recovering sparse representations. One significant implication is that under quite general conditions, the combinatorial problem of finding sparse solutions to systems of linear equations can be efficiently and exactly solved via convex optimization, by minimizing the l^1 norm [77].

Some results in sparse signal representation suggest that the linear relationships among high-resolution signals can be accurately recovered from their low-dimensional projections [78]. Although the SR problem is very ill-posed, making precise recovery impossible, the image patch sparse representation demonstrates both effectiveness and robustness in regularizing the inverse problem. Following this idea, Yang and Wright et al. [79] adopted sparse representation for SR on face images. Similar to the learning-based methods, they rely on patches from the input image. However, instead of working directly with the image patch pairs sampled from high and low-resolution images, they learned a compact representation for these patch pairs to capture the co-occurrence prior, improving the speed of the algorithm.

Two constraints are modeled in this work: 1) reconstruction constraint, which requires that the recovered X (higher-resolution image recovered from the SR method) should be consistent with the input Y (low-resolution image)with respect to the image observation model; and 2) sparsity prior, which assumes that the high resolution patches can be sparsely represented in an appropriately chosen over-complete dictionary, and that their sparse representations can be recovered from the low resolution observation.

1. Reconstruction constraint. The observed low-resolution image Y is a blurred and down-sampled version of the high resolution image X:

$$Y = SHX, \tag{17}$$

here, H represents a blurring filter, and S the down-sampling operator. SR remains extremely ill-posed, since for a given low-resolution input Y, infinitely many high-resolution images X satisfy the above reconstruction constraint. We further regularize the problem via the following prior on small patches x of X:

2. Sparsity prior. The patches x of the high-resolution image X can be represented as a sparse linear combination in a dictionary D_h trained from high-resolution patches sampled from training images:

$$\times \approx D_h \alpha \qquad \textit{for some } \alpha \in R^k \textit{ with } \|\alpha\|_0 \ll K. \tag{18}$$

The sparse representation α will be recovered by representing patches y of the input image Y , with respect to a low resolution dictionary Dl co-trained with Dh.

Inspired by their work, Bilgazyev et al. [80] introduced the use of a Dual Tree Complex Wavelet Transform (DT-CWT) in a sparse representation framework for SR reconstruction. They estimate the high-frequency components, rather than studying the direct relationship between the high and low-resolution images. They showed that, using conventional features such as Eigenfaces and facial parts, the proposed algorithm achieves much higher recognition accuracy on face images with variation in either illumination or expression. They reported that the recognition rates outperformed SRSR [79] and S2R2 [2] for the CMU PIE database.

One of the limitations of the sparse representation is the assumption of pixel-accurate alignment between the test image and the training set. This leads to brittleness under pose and misalignment, making it inappropriate for real applications. In [81] they show an approach to rectify this weakness while still preserving the conceptual simplicity and good recognition performance of sparse representation.

6 Comparison between Principal Approaches

It is important to evaluate various LR FR methods based on certain evaluation criteria to have a clear idea on their behavior. Nowadays, LR FR methods are evaluated based on HR FR criteria and databases due to the lack of general criteria and databases originally developed for LR FR.

There is currently no database for LR FR, so that for evaluations on most of the existing LR FR methods, face images with frontal view, neutral expression, and illumination variations are selected and preprocessed such as down-sampling and blurring instead of the actual LR images taken by surveillance cameras. At present, the widely used databases for LR FR are FERET/Color FERET [85], CMU PIE/CMU Multi-PIE, FRGC, and SCface. Other databases such as XM2VTS [82], UMIST [83], ORL [84], and KFDB [85] are also used to evaluate LR FR methods.

The performances of LR FR methods tested on some databases mentioned above are summarized in figures 5, 6 and 7. FERET database mainly covers expression variation; which is relatively old compared with other databases. Tested on FERET, resolution-robust feature representation methods such as CLPMs slightly outperform recognition-oriented SR methods such as S2R2. Then, FERET is more suitable for simple conditions, such as single expression variation.

FRGC and SCface are two relatively complicated databases in which recognition oriented SR methods obtain much better performance, although the resolution-robust feature representation method MDS tested on SCface is greatly superior to DSR. This result is attributed to the use of frontal view probe images.

In addition, the evaluations on CMU PIE or CMU Multi-PIE demonstrate that recognition-oriented SR methods such as S2R2 and DSR are more easily against unconstrained variations, e.g., illumination and expression. However, the resolution-robust feature representation method CKE also obtains promising results on CMU Multi-PIE mainly with the help of the kernel trick.

It is very difficult to rank all the methods based on the existing and widely used image-based standard databases. Also, no method can satisfactorily handle the LR problem in face recognition under all complicated variations. For example, M2TSR is the only method specially designed to deal with pose and illumination problem in LR classification. However, its performance on the FERET database with 14 × 9 is only 74.6 %, which is still far below the requirement of practical use. The performances of different methods depend on different databases to some extent. Therefore, a few standard LR face databases are necessarily built for fair comparisons.

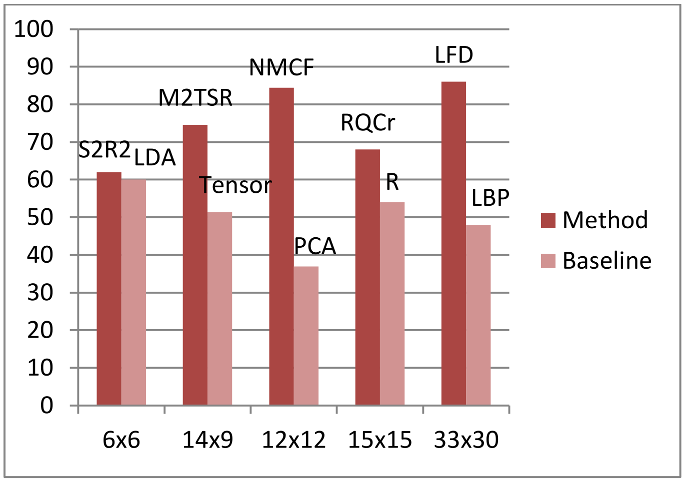

Fig. 5. Performance of principal and baseline methods on FERET database.

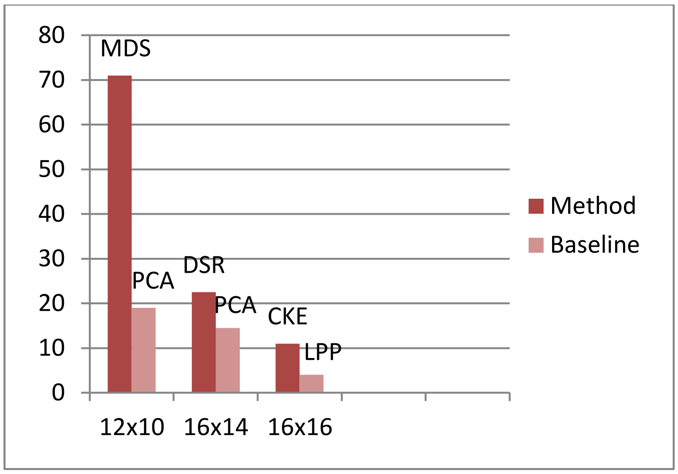

Fig. 6. Performance of principal and baseline methods on SCFace database.

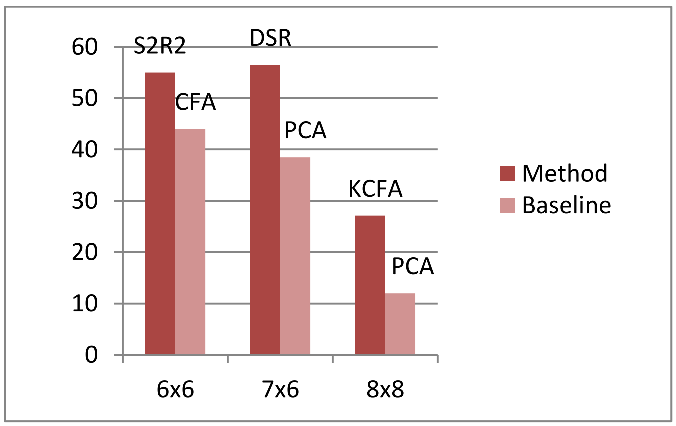

Fig. 7. Performance of principal and baseline methods on FRGC database.

Other databases based on video-based real environments allow evaluating the performances of some LR FR methods such as CLEAR2006 [86]. Recently, some researchers attempted to build LR databases to simulate real world. Yao et al. [87] created a face video database, UTK-LRHM, obtained from long distances and with high magnifications, both indoors and outdoors under uncontrolled surveillance conditions. Also, they developed a wavelet transform based multi-scale processing algorithm, which was successful in improving recognition rate.

Another attempt was a database named “labeled faces in the wild” (LFW), [88] containing images that were collected from the web. Although it has natural variations in pose, illumination, expression, etc., there is no guarantee that such a database can accurately capture all variations found in the real world. Besides, most objects in LFW only have one or two images, which might not be enough to conduct different face recognition experiments. That is to say, there is still no LR benchmark database for public comparisons at present.

In [20] they constructed a video-based face database with uncooperative subjects in an uncontrolled indoor environment. They designed three group experiments for evaluating the effects of distance (resolution), illumination, and misalignment on the abilities of the two methods for dealing with the LR FR problem. In 2.0 meters distance, S2R2 and CLPMs obtain the promising identification accuracies (IDA vs. Rank-1) of 80 % and 87.5 %. It is noticeable that S2R2 without alignment is even inferior to the LR case with alignment. Thus alignment is very important for face recognition, especially for LR FR.

From the results of many experiments reported in the literature on the real-world environment, the existing LR FR methods have not performed well under real-world scenarios. The representatives of LR FR methods such as S2R2 and CLPMs are severely affected by complicated conditions, e.g., distance, illumination, misalignment. In such cases, compared with S2R2, the performance of CLPMs is much poorer, which is probably attributed to no stable features involved in CLPMs. However, S2R2 is also unsuitable for real-time application mainly due to the complication of the model parameter learning. Briefly, the existing LR FR methods should be greatly improved so as to be used for real applications.

Based on the evaluations on image-based standard databases and video-based real environment, we can have an idea on the performances of the existing LR FR methods. Misalignment and some environmental conditions (e.g., illumination, distance, noise) and face variations (e.g., expression, pose) can bring a lot of effects on LR FR.

7 Conclusions

LR is a challenging and interesting subarea in face recognition. After introducing the LR FR concept and analyzing some representative methods, we can conclude some ideas discuses above related to the different approaches:

- It is necessary to create standard databases and criteria to evaluate LR FR methods performances, since they have to lead with databases and metrics originally designed for HR FR.
- Feature SR outperforms image SR from recognition perspective.
- The first SR techniques based on reconstruction, represent an intuitive approach to improve a face image, but mostly for a visual improvement because they are not designed from a recognition point of view.
- To match high resolution training images with low-resolution probe images, the down-sampling approach in the low-resolution domain have shown to be better than applying SR, especially when faces are of very low resolution.
- Resolution-robust feature methods are a recent and promising way to aboard LR approach, trying to obtain a unified feature space to solve the dimensional mismatch problem.
- Sparse representation and coupled metric learning are two representatives resolution-robust feature methods with the advantages of low computational complexity and lower requirement of training samples, making it more suitable for real applications.

- There is no an unique and better way for face image SR with recognition purpose since all LR FR methods have advantages and disadvantages related to particular properties of each one.

8 References

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