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CENATAV
Centro de Aplicaciones de Tecnologías de Avanzada
MINISTERIO DE LA INDUSTRIA BÁSICA

REPORTE TÉCNICO
Reconocimiento de Patrones

SERIE AZUL

# Fingerprint Reconstruction and Orientation Field Estimation: A Review

**Armando Rodríguez Fonte and José Hernández Palancar**

**RT_070** enero 2015

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# Fingerprint Reconstruction and Orientation Field Estimation: A Review

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**Abstract.** The fingerprint is one of the most commonly used biometric characteristics for identification of individuals. Because they are unique and immutable. Most existing systems are based on minutiae extraction. Recently, the features obtained from ridges specific details such as: dots, pores and protrusions have received more attention because of its proven contribution to increased efficacy in comparison with latent fingerprints. In several cases latent does not have the desirable quality and the ridges are broken or simply lost. For those images it is necessary to reconstruct the orientation field and regenerate the ridges structures. In this review we summarize the main methods for orientation field estimation and fingerprint reconstruction and synthesis. We mention the utilities of those methods, we make several comparisons between the methods and we discuss the conclusions of this research.

**Keywords:** fingerprint images, quality, orientation, fingerprint synthesis, fingerprint reconstruction.

**Resumen.** Las huellas dactilares son uno de los rasgos biométricos más comúnmente usados para la identificación de las personas. Esto se debe a que son únicos e inmutables. La mayoría de los sistemas existentes se basan en la extracción de rasgos conocidos como minucias, presentes en las huellas. Recientemente, las características obtenidas de las crestas tales como: puntos, poros y protuberancias han recibido mayor atención por su gran contribución al incremento de la eficacia en la comparación de impresiones latentes. En muchos casos las huellas latentes no cuentan con la calidad deseada, sus crestas aparecen truncadas o simplemente perdidas. De igual forma ocurre en imágenes que presentan zonas de mala calidad donde es necesario reconstruir el campo de orientación y regenerar las estructuras de las crestas. En esta investigación se hace un resumen de los principales métodos para estimar el campo de orientación y la reconstrucción y síntesis de impresiones y huellas dactilares. Se mencionan las utilidades de estos métodos, se realizan comparaciones entre ellos y se discuten las conclusiones para de esta investigación.

**Palabras clave:** huella dactilar, calidad, reconstrucción de huellas dactilares, orientación.

## 1 Introduction

Fingerprints as a kind of human biometrics have been widely used for people recognition in commercial and forensic areas due to its uniqueness and immutability [10]. Its applications have gone beyond the criminal issues to a mean of assuring individual rights. Former fingerprint analysis techniques (highly reliable contextual analysis) oriented to identification have evolved around off-line applications (large database searches, high capacity computers) related to law enforcement operations. On the other hand, newer security uses of automatic fingerprint identification systems (AFIS) involving verification, such as banking, shopping or access control, require real-time operations with low cost. For these cases it is important to develop fast fingerprint analysis techniques that still retain the characteristics of the fingerprint image [10].

Two levels of description are usually defined for fingerprints: a global or topological classification, based on the typical patterns formed by ridges (arcs, loops, whorls) and a local or detailed description based on ridge endings and bifurcations (minutiae).

The first level is the global level, the ridge line flow delineates a pattern. Singular points, called loop and delta, act as control points around which the ridge lines are wrapper [15]. Singular points and coarse ridge line shape are useful for fingerprint classification and indexing, but their distinctiveness is not sufficient for accurate matching.

The second level is the local level, a total of 150 different local ridge characteristics, called minute details, have been identified [16]. The two most prominent ridge characteristics, called minutiae are: ridge endings and ridge bifurcations. A ridge ending is defined as the ridge point where a ridge ends abruptly. A ridge bifurcation is defined as the ridge point where a ridge forks or diverges into branch ridges. Minutiae in fingerprints are generally stable and robust to fingerprint impression conditions. Although a minutiae-based representation is characterized by a high saliency, reliable automatic minutiae extraction can be problematic in extremely low-quality fingerprints devoid of any ridge structure [10].

Level three is the very-fine level, it comprises features obtained from ridges specific details such as: dots, pores, protrusions, etc [10]. These traits begin to receive more attention now due to its contribution to increased accuracy in comparison of latent fingerprints. In several cases the latent has not the desirable quality and the ridges are broken or simply lost. For those images it is necessary to reconstruct the orientation field and to regenerate the ridges structures.

Several methods for fingerprint analysis work with segmented representations of the image, looking for singular points and minutiae in the context of the global flow of patterns. From all the biometric techniques, automatic fingerprint based systems are regarded as the most popular and reliable for automatic personal identification. With the increasing attention on automatic identity verification, fingerprint recognition systems have become a popular research topic over the last decades. However, there still exist critical research issues such as the low accuracy in the processing of poor quality images.

In practice, the quality of an acquired fingerprint image can easily be degraded by various factors such as wet or dry finger, dirty or greasy finger, wounded finger and finger with scars or creases. Very often, these factors are hard to avoid in practical operations. Even with high quality imaging sensor under well controlled environment, the acquired fingerprint image could be of low quality. The key for solving this problem by most practical systems is to enhance the fingerprint image before the processes of feature extraction and fingerprint matching. Instead of directly employing image enhancement generic methods for improving the fingerprint image quality, most fingerprint enhancement methods are based on the characteristic structure within the fingerprint, which have been proven to be more effective in practice. One of the most important features of a fingerprint is the pattern of the ridge flow and orientation-based enhancement methods are very used in automatic fingerprint recognition systems.

In this review we summarize the main methods for orientation field estimation and fingerprint reconstruction and synthesis. We explained the advantages and drawbacks of those, we make several comparisons between the methods and after that we discuss the conclusions for this research.

The review is organized as follows: in Section 2 methods for orientation field estimation are analysed. This section is divided in gradient based methods and model based methods. In Section 3 methods for fingerprint reconstruction and synthesis are studied, that section is divided in two parts too.

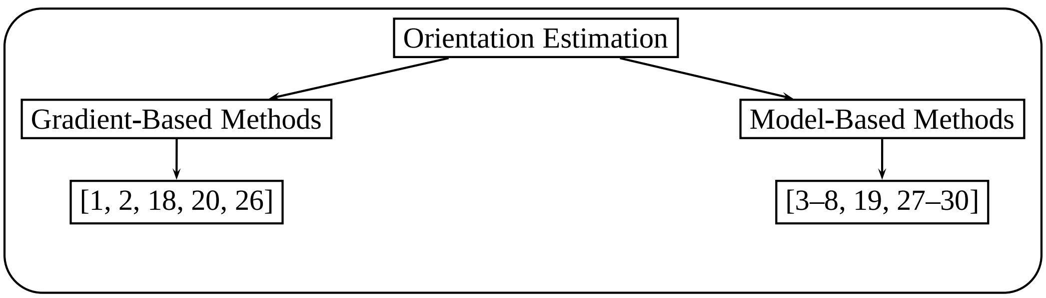
## 2 Orientation Estimation

Orientation field is defined as the local direction of the ridge-valley structures. As a global feature, plays a very important role in fingerprint classification and identification. An orientation field can effectively summarize the information contained in a fingerprint pattern and it is a rich information resource for fingerprint features retrieval and processing.

The values of orientation angles of fingerprint pattern have a critical impact on almost all subsequent processes in automatic fingerprint recognition systems. Orientation field has been widely used for fingerprint image enhancement [17–21], singular points detection [2, 22] and classification [23–25].

Orientation field estimation methods can be divided in two categories: gradient-based methods and model-based methods.

Orientation Estimation Taxonomy




### 2.1 Gradient-Based Method

One of the most popular approach to estimate the orientation image in fingerprint is the gradient method introduced by Kass and Witkin in 1987 [1]. The most important advantage of this algorithm is the fact that the obtained values are continuous. The main steps of the algorithm are as follows:

1. Divide the fingerprint image into blocks of size w $\times$ w
2. Compute the gradients $\partial_x$(i,j) and $\partial_y$(i,j) at each pixel (i,j). Depending on the computational requirements, the gradient operator may be a simple *Sobel* operator or the more complex *Marr-Hildreth* operator.
3. Estimate the local orientation of each block centred at pixel (i,j) using the followings equations:

$$V_x(i,j) = \sum_{u=i-\frac{w}{2}}^{i+\frac{w}{2}} \sum_{v=j-\frac{w}{2}}^{j+\frac{w}{2}} 2\partial_x(u,v)\partial_y(i,j), \tag{1}$$

$$V_y(i,j) = \sum_{u=i-\frac{w}{2}}^{i+\frac{w}{2}} \sum_{v=j-\frac{w}{2}}^{j+\frac{w}{2}} (\partial_x(u,v)^2 - \partial_y(i,j)^2), \tag{2}$$

$$\Theta(i,j) = \frac{1}{2}tan^{-1}1(\frac{V_y(i,j)}{V_x(i,j)}), \tag{3}$$

where $\Theta$(i,j) is the least square estimate of the local ridge orientation at the block centred at pixel (i,j).

Hong et al. [18] proposed some improves for this gradient method. Due to the presence of noise, the estimation of local orientation $\Theta$(i,j) may not always be correct. They used a low pass filter to modify the incorrect local ridges orientation.

In the paper of Hong they use w = 16 but conducted experiments show that the size of w $\times$ w blocks should be at least twice the average distance between the fingerprint ridges [20].

Bazen and Gerez [2] presented a new method to estimate the directional field from the gradients and the coherence in any pixel location, which is based on Principal Component Analysis (PCA). PCA computes a new orthogonal base given a multidimensional date set such that the variance of the projection on one of the axes of this new base is maximal, while the projection on the other one is minimal. They proved that this method provides exactly the same results as the averaged square-gradient method.

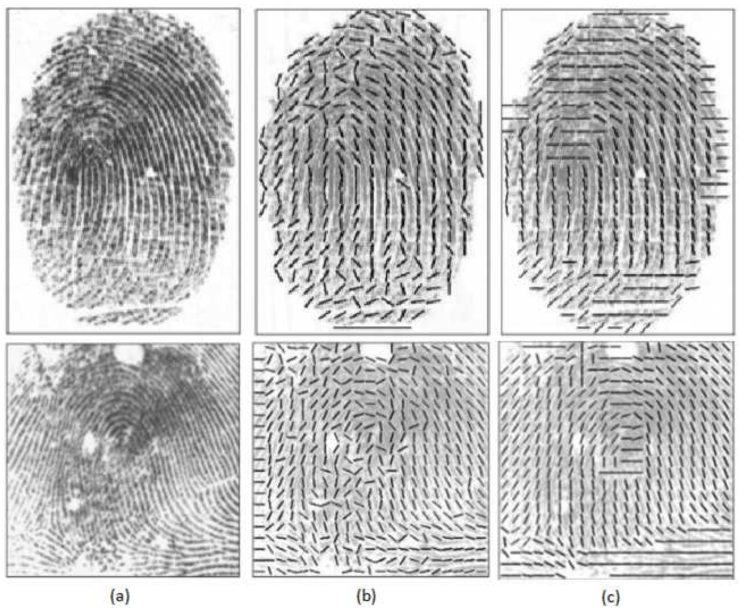



**Fig. 1.** Some comparative examples for gradient methods. (a) Original fingerprint. (b) Kass and Witkin [1]. (c) Bazen and Gerez [2].

The method [1] estimates the orientation field with a lot of noise for poor-quality fingerprints (see figure 1). The Bazen and Gerez method [2] is better but it is heavily influenced by noise such as creases and scars.

Wieclaw [20] suggested some modification for the original algorithm [1]: when the computed gradients values are the same $\partial_x$(i,j) = $\partial_y$(i,j) then they add randomly $\pm 1$ to one of the gradients. If one of the gradients values is equal to 0 (for example $\partial_x$(i,j) = 0) then they add also randomly $\pm 1$.

Mei et al. [26] proposed a systematic gradient-based method for computing orientation field in fingerprints. They mentioned two shortcomings for gradient-based methods:

1. The basic steps for almost all gradient-based methods are: point gradient vector computing and block gradient vector computing. The choice about to apply normalization or not, is consider a problem of the first. One problem of the second is the selection of the scale for the block, small scale is beneficial to accuracy but sensitive to noise, while large scale is more resistant to noise, but the accuracy comes down.
2. For poor fingerprint images, especially for images with large scale noise, the results of these method are still not satisfactory.

In their algorithm they included the basic steps for gradient-based methods. They did an analysis about the necessity of normalization for the point gradient vectors and how to choose the suitable scale for the block. After that they presented one step for noise region marking (see figure 2(a),(c) and (g)) and another step for orientation field prediction for the noise region (see figure 2(b),(d) and (h)) These two steps improving the robustness against noise.

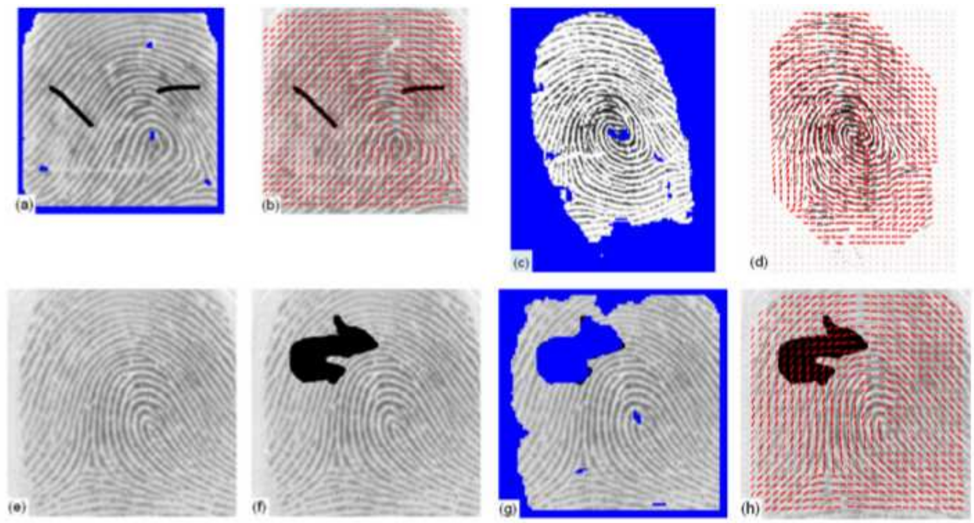



**Fig. 2.** The marked noise region and predicting results. (a) The marked noise region for a fingerprintf; (b) the predicting results for (a); (c) the marked noise region for a fingerprint; (d) the predicting results for (c); (e) the original image; (f) image with artificial noise; (g) the marked noise region for (f); (h) the predicting results for (g).

**Table 1.** Existing Gradient-Based Methods for Orientation Field Estimation.

| Method | Year |
|---|---|
| Kass and Witkin [1] | 1987 |
| Hong et al. [18] | 1998 |
| Bazen and Gerez [2] | 2002 |
| Wieclaw [20] | 2011 |
| Mei et al. [26] | 2012 |

## 2.2 Model-Based Methods

More complex methods for orientation field estimation are model-based methods. These rely on the global regularity of orientation values around the singular points.

First work in this direction was the zero-pole model presented by Sherlock and Monro [3], they showed that local ridge orientation of fingerprint can be described using the direction field concept of differential geometry. They presented also a simple model of fingerprint local ridge orientation topology in terms of the positions of cores and deltas. This method is known in the literature as *zero-pole model*.

Vizcaya and Gerhardt [4] had made an improvement using a piecewise linear approximation model around singular points to adjust the behavior of zero-pole. The neighborhood of each singular point is divided in eight regions and the influence of the singular point is assumed to change linearly in each region.

Zhou and Gu critiqued these two models in their papers [27] and [5]. They explained that the influence of a singular point is the same for any point on the same central line, so they are effective only near the singular points and they lack of accuracy far from singular points. Furthermore, both the two models cannot deal with plain arch (fingerprints which have no singular points). In a word, these two models could not meet the need of real applications. As a result these researchers proposed a combination model to represent the fingerprint orientation field. They define a polynomial model to globally represent the orientation field and they use a point-charge model to improve the local accuracy of estimation at each singular point ([27],[5]). The value of a orientation field is always defined within [0,pi), therefore the orientation field cannot be modelling directly. A solution to this problem is to map the orientation field to a continuous complex function [4]:

$$U = RE + iIM = \cos(2\theta) + i\sin(2\theta), \tag{4}$$

where RE and IM denote respectively the real part and imaginary part of the complex function, U(x, y). RE(x, y) and IM(x, y) are continuous with x, y in those regions. The above mapping is a one-to-one transformation and $\theta$(x, y) can be easily reconstructed from the values of RE(x, y) and IM(x, y).

Near to singular points, the orientation is no longer smooth, so it is difficult to model with a polynomial function. A model named point-charge (PC) is added at each singular point. Compared with the model provided in [3] the point charge-model uses different quantities of electricity to describe the neighborhood of each singular point instead of the same influence at all singular points. The influence of a standard (vertical) core at the point, (x, y), is defined as:

$$PC_{Core} = H_1 + iH_2 = \begin{cases} \frac{y-y_0}{r}Q - i\frac{x-x_0}{r}Q, & r \le R, \\ 0, & r > R, \end{cases} \tag{5}$$

where (x0, y0) is this core's position, Q is the quantity of electricity, R denotes the radius of its effective region, and $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$. The radius of a standard delta is:

$$PC_{Delta} = H_1 + iH_2 = \begin{cases} -\frac{y-y_0}{r}Q - i\frac{x-x_0}{r}Q, & r \le R, \\ 0, & r > R. \end{cases} \tag{6}$$

To combine the polynomial model (PR, PI) with Point-Charge smoothly, a weight function is defined. For Point-Charge, its weight at (x, y) is defined as:

$$\alpha^k_{PC}(x,y) = 1 - \frac{r^k(x,y)}{R^k}, \tag{7}$$

where $(x^k_0, y^k_0)$ is the coordinate of the k-th singular point, $R^k$ is its effective radius, and $r^k$ is set as min($\sqrt{(x-x^k_0)^2 + (y-y^k_0)^2}, R^k$).

For polynomial model, its weight at (x, y) is:

$$\alpha_{PM}(x,y) = max\{1 - \sum_{k=1}^{K} \alpha^k_{PC}, 0\}, \tag{8}$$

where K is the number of singular points.

The examples show how the combination model can reconstruct the orientation field smoothly and accurately even with the existence of noise (figures 3(d) and 4(d)). The zero-pole model can only roughly describe the orientation (figures 3(b) and 4(b)) and the piecewise linear model is better near singular points but it fails far from these zones (figures 3(c) and 4(c).

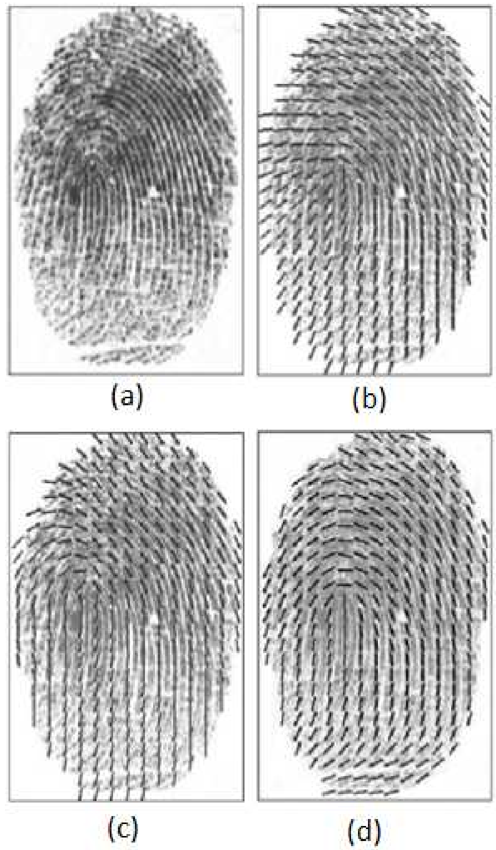



**Fig. 3.** Some comparative examples for gradient methods. (a) Original plain fingerprint. (b) Zero-pole model [3]. (c) Piecewise linear model [4]. (d) Combination Model [5].

Chikkerur et al. [19] proposed an algorithm based on Fourier analysis to extract local ridge orientation, ridge frequency and ridge quality measure. With the exception of the singularities such as Core and Delta any local region in the fingerprint image has a consistent orientation and frequency. Therefore, the local region can be modelled as a surface wave, it is characterize by its orientation $\theta$ and frequency *f*. The parameters of the surface wave ($f$, $\theta$) may be easily obtained from its Fourier spectrum that consists in two impulses whose distance from the origin indicates the frequency and its angular location indicates the orientation of the wave [19]. The surface wave model is an approximation. The Fourier spectrum of a real fingerprint image is characterized by a distribution of energies across all frequencies and orientations. They used a probabilistic approximation to estimate the orientation for each block. They represented the Fourier spectrum in polar form as F(r,$\theta$), they defined a probability density function f(r, $\theta$) and the marginal density functions f(r), f($\theta$) as:

$$f(r,\theta) = \frac{|F(r,\theta)|^2}{\int_r \int_\theta |F(r,\theta)|^2 \, d\theta \, dr}, \tag{9}$$

$$f(\theta) = \int_r f(r,\theta) \, dr, \; f(r) = \int_\theta f(r,\theta) \, d\theta. \tag{10}$$

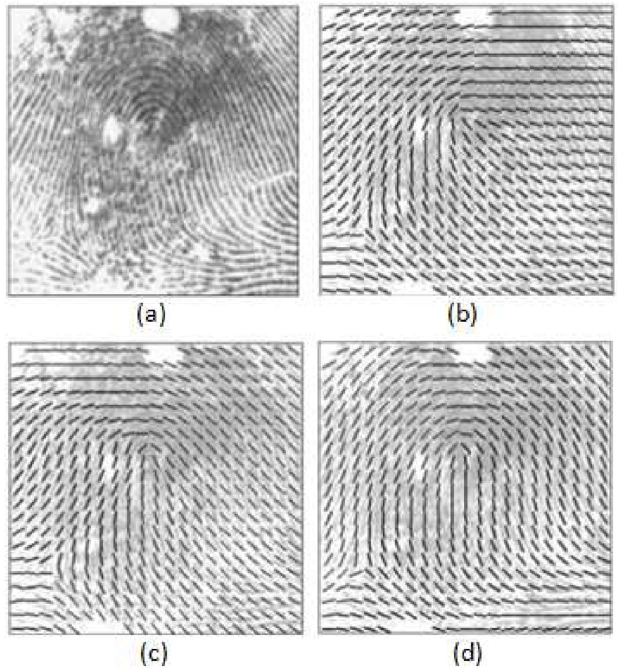



**Fig. 4.** Some comparative examples for gradient methods. (a) Original rolled fingerprint. (b) Zero-pole model [3]. (c) Piecewise linear model [4]. (d) Combination Model [5].

They assumed that the orientation $\theta$ is a random variable that has the probability density function f($\theta$). The expected value of the orientation may be obtained by:

$$E\{\theta\} = \int_r r * f(r)\, dr. \tag{11}$$

An important and cited Fourier-based algorithm for orientation estimation was presented by Wang et al. [6]. The name of the algorithm is Fingerprint Orientation Model Based on 2D Fourier Expansion (FOMFE).

An orientation field can be fully represented by a discrete matrix whose elements represent the local average directions of fingerprint ridges [6]. In order to avoid the difficult problem created by orientation discontinuity $(\Pi \longleftrightarrow 0)$, a popular approach is to map the orientation field into a new vector field where each orientational element is denoted as a 2D vector $v = (v_s, v_c)$ with $v_s, v_c$ being the phase functions of $cos2\theta$ and $sin2\theta$, respectively and $\theta$ is the orientation angle computed by [1].

Since ridges and valleys alternate throughout fingerprint areas, it is nature to assume that fingerprint orientation will behave in a periodic manner. Therefore, it is reasonable to use FOMFE which is a set of cosine and sine functions to represent fingerprint orientation field. For a bivariate function $f(x, y)$ in a restricted area in $R^2$ say $(-l \leqslant x \leqslant l, -h \leqslant y \leqslant h)$, its 2D Fourier expansion can be expressed in the following form [31]:

$$f(x, y) = \sum_{m=0}^{k} \sum_{n=0}^{k} \Psi(mvx, nwy, \beta_{mn}) + \varepsilon(x, y), \tag{12}$$

where $m, n \in N$; $\varepsilon(x,y)$ is the residual; the fundamental frequencies: $v = \frac{\Pi}{l}$ and $w = \frac{\Pi}{h}$, are on the orthogonal $x$ and $y$ axes; and

$$\begin{aligned}\Psi(mvx, nwy, \beta_{mn}) = \lambda_{mn}[&a_{mn}cos(mvx)cos(nwy)\\ &+b_{mn}sin(mvx)cos(nwy)\\ &+c_{mn}cos(mvx)sin(nwy)\\ &+d_{mn}sin(mvx)sin(nwy)],\end{aligned}$$

where $\lambda_{mn}$ is a constant scalar which can be found in [6]. $\beta_{mn}$ is actually composed of four Fourier coefficients $(a_{mn}, b_{mn}, c_{mn}, d_{mn})$ to be estimated. They can use two bivariate trigonometric polynomial functions $f_c(v_c)$ and $f_s(v_s)$. The specific problem in FOMFE is to find the coefficient matrices $(a_{mn}, b_{mn}, c_{mn}, d_{mn})$. The problem can be formulated as a classical linear least square(LSQ) problem [32].

The Authors of FOMFE compared their methods with a classical gradient-based method [1] using image with different levels of quality (see figure 5). Results showed how FOMFE is better than [1] in poor quality images.

The main advantages of this model are:

1. FOMFE does not need to get singular points data (location and type) from others detection methods.
2. It can compute fingerprint global features including singular points areas.
3. This method can improve the accuracy of fingerprint feature extraction (especially in poor quality images) and, thus, improve fingerprint matching.
4. FOMFE has also been applied to singular points retrieval and continuous fingerprint indexing.

Huckemann et al. [28] were inspired by Sherlock and Monro models [3] and they added global features present across all classes of fingerprints, such as parallel ridges near the joint and circular ridges at the fingertip modelled using quadratic differentials (QDs).

They mentioned in their paper several desirable properties for a model of orientation field estimation:

1. Accuracy: The model should describe the true orientation field as much as possible.
2. Invariance under euclidean motions: Only parameters that are invariant under rotations and translations of the fingerprint image can serve as database indexes.
3. Robustness against partial observation: There are cases where many images for the same fingerprint do not show the same regions, that is why it is important to have robust parameters in the model against changes of the fingerprint regions.
4. Low dimension: Increasing the number of parameters will most likely decrease the reliability of estimates of single parameters.
5. Interpretability: Parameters should have a geometrical meaning, i.e., they should be identifiable and serve to explain the features of the model.
6. Predictive power. It should be possible to predict bad quality, noisy, or unobserved regions, i.e., to interpolate or even extrapolate.

This model can approximate fingerprint orientation field using only five parameters. These parameters are geometrically interpretable and have a clear meaning.

Gottschlich et al. [7] presented a novel method for orientation field estimation . The line-sensor based method traces ridge and valley lines. It builds a coherent structure of locally parallel line segments from which the OF estimation is derived. To this end, the line-sensor based approach makes good use of a property inherent in fingerprints: the continuity of ridge and valley flow perpendicular to the flow. This is


**Fig. 5.** Comparison between FOMFE and gradient method [1]. (a)Top-down: Three impressions of the same fingerprint with image quality going from good to moderate to bad, (b) Gradient method propoused in [1], (c) Orientation field estimation by FOMFE [6]. Taken from [6].

a multiscale approach, since, at first, line segments are discovered locally. In a second step, neighboring parallel line segments will be merged and eventually they broadcast their orientation to a medium-scale vicinity. By this means, the orientation field is constructed. The general structure of the algorithm is as follow (see Figure 6 for an illustration of the main steps):

1. The grayscale image is smoothed, binarized and morphologically improved.
2. A rudimentary line tracing detects ridges and valleys, completely or partially.
3. The discovered line pieces are analyzed for parallel pieces in a neighborhood orthogonal to the line piece.
4. They take the parallelism of line pieces into account and group them to parallel structures; the larger the structure, the higher the confidence in the resulting orientation estimation. All structures that cover a minimum number of pixels are merged, and all line pieces propagate their orientation orthogonally. In this way, the orientation field is estimated.
5. Missing blocks are iteratively reconstructed until the orientation field is complete. See Figure 7 for an illustration of the main steps.

Tao et al. [29] presented a new approach for orientation field estimation. They were motivated by FOMFE [6] and they remarked two drawbacks of FOMFE:

1. FOMFE is sensitive to abrupt changes in orientation field.

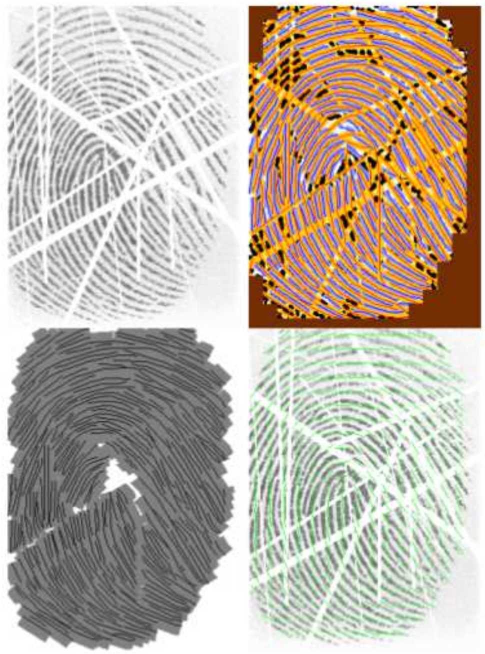


**Fig. 6.** The main steps of the line-sensor based method. Top left: fingerprint image with 15 randomly added artificial scars; top right: traced ridges (blue) and valleys (red); bottom left: merged coherent structure; bottom right: derived orientation field after reconstruction. Taken from [7].

2. FOMFE does not consider that blocks of different quality should have different impacts on the model.

Thus, they proposed a novel technique for dealing with the two drawbacks by fingerprint orientation model-based on weighted 2D fourier Expansion(W-FOMFE):

1. They adopted a gradient-based method [2] to compute the original orientation field $\Theta$.
2. Calculate the Harris-corner strength(HCS) [33].
3. Then use the HCS to remove abrupt changes in orientation field.
4. Finally, incorporate the normalized Harris-corner strength as weighted value into original FOMFE.

Hou et al. [30] proposed a framework for modelling the fingerprint orientation field based on the variational principle, where the orientation pattern can be estimated through solving the associated Euler-Lagrange equation.

Variational principle is an important method in physics for determining the state or the dynamics of a physical system. It was originated by Leibniz and founded by Euler and Lagrange. Variational principle seeks the solution through finding the extremum (minimum, maximum or saddle point) of a function. The method can be expressed using the calculus of variations, which is a branch of mathematics dealing with integral minimization. The function to be minimized can be formed as an integral involving unknown function $f$ or its derivatives as follows:

$$J[f] = \int_{X_1}^{X_2} L(X, f, f')dx. \tag{13}$$

Then the problem is to find the extremal function $f^*$ where the rate of change of the functional J[$f$] is zero.

The main steps of the proposed fingerprint orientation modelling method are summarized as follows:

1. Preprocessing: The input image is segmented into foreground (image region containing fingerprint structures) and background. A simple method [18] is based on the variance of a block. If the variance is small, that block will be marked as background, otherwise, it will be foreground.
2. Initialization: The fingerprint orientation field $\theta$ is preliminarily estimated using a local method. The method by Kass and Witkin [1] is employed in this study, where the orientation y is projected into the vector space according to:
$$\Theta = (cos2\theta, sin2\theta). \tag{14}$$
3. PDE processing: For the vector representation of the orientation field (by 14), each component is processed using a procedure described by authors in [30]. The iteration is terminated till convergence or maximum iteration number (they use 10 in their experiments).
4. Output: The final orientation is reconstructed from the vector representation by $\varphi = 0.5 * arctan(v/u)$ (the process to obtain v and u is explained in the paper [30]).

They did several comparisons with FOMFE in terms of singular points detection and matching.

They built a ground truth for comparisons of singular points extraction using all fingerprint images in FVC 2004 Db1a. They manually processed and labelled core and delta points. To compare both methods, after the end of orientation modelling, the Poincare index method is applied to the orientation field for extracting the core and the delta point. The comparisons showed an evidently superiority of this method over FOMFE. They obtained a less number of false positives and negative. Therefore they have most precision. We detect one problem in this comparison: the fact to use FOMFE for detecting singular points with Poincare Index, because FOMFE can detect singular points for itself.

The matching comparisons were done using NIST fingerprint software [34]. The original version of FOMFE was used and also it was executed using the FOMFE and the proposed method [30] in the orientation field estimation step. FOMFE method and the proposed method [30] had similar results for almost all experiments. They presented high accuracy and their results were considerably better than original NIST.

Feng et al. [8] believe that the major limitation of conventional orientation field estimation algorithms is that they do not adequately incorporate prior knowledge of fingerprints. Then, they presented an algorithm using prior knowledge of fingerprint orientation field.

They explained an analogy between a fingerprint orientation field and a sentence in a natural language. A sentence is comprised of words which are further comprised of letters. Similarly, a fingerprint orientation field is comprised of orientation patches which are further comprised of orientation elements. Hence, a fingerprint orientation field can be viewed as a sentence, an orientation patch can be viewed as a word, and an orientation element can be viewed as a letter. Spelling correction in a sentence is possible because not all possible combinations of letters are valid to form words and not all possible combinations of words form a valid sentence. Similarly, error correction in orientation fields is possible because not all possible combinations of orientation elements are valid and not all possible combinations of orientation patches are valid for a fingerprint. Spelling correction techniques use dictionary and context information to detect and correct spelling errors.

They proposed an orientation field estimation algorithm. It was inspired by the spelling correction method. They first build a dictionary of reference orientation patches using a set of orientation fields extracted from real fingerprints. Given an input fingerprint, they estimate an initial orientation field using traditional orientation field estimation approaches. For poor quality fingerprints, such as most latents, the initial orientation fields are very noisy. Errors in the initial orientation field need to be corrected using dictionary as well as context information. Specifically, for each initial orientation patch, they find a list of candidates from the dictionary which might be the true orientation patch. Contextual information is then used to determine a single candidate for each patch.

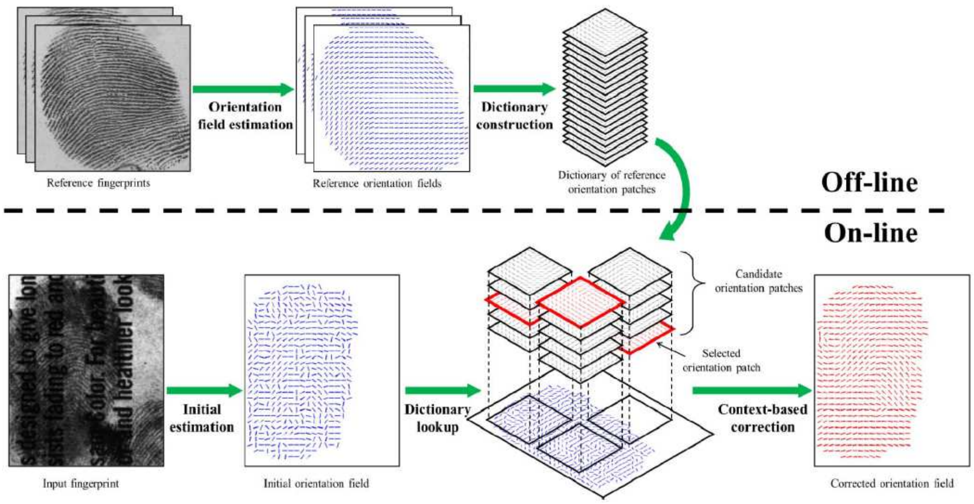



**Fig. 7.** The proposed system consists of an offline dictionary construction stage and an online orientation field estimation stage. Taken from [8].

The proposed orientation field estimation algorithm consists of an offline dictionary construction stage and an online orientation field estimation stage (see figure 7). In the offline stage, a set of good quality fingerprints of various pattern types (arch, loop, and whorl) is manually selected and their orientation fields are used to construct a dictionary of orientation patches. In the online stage, given a fingerprint image, its orientation field is automatically estimated using the following steps:

1. Initial estimation. The initial orientation field is obtained using a local orientation estimation method such as local Fourier analysis [35].
2. Dictionary lookup. The initial orientation field is divided into overlapping patches. For each initial orientation patch, its six nearest neighbors in the dictionary are viewed as candidates for replacing the noisy initial orientation patch.
3. Context-based correction. The optimal combination of candidate orientation patches is found by considering the compatibility between neighboring orientation patches.

**Table 2.** Number of parameters used for several methods. K denotes the number of singular points; the locations of the singular points have not been counted as parameters as they are extracted from the image.

| Model | Number of Real Parameters |
|---|---|
| Sherlock and Monro [3] | 1 |
| Vizcaya and Gerhardt [4] | 10K |
| Zhou and Gu [27] | 13 |
| Gu et. al [5] | k + 32 |
| Wang et al. (FOMFE) [6] | 4 |
| Huckemann et al. [28] | 5 |
| Gottschlich et al. [7] | 7 |

## 2.3 Conclusions for Orientation Field Estimation

We study the most used methods for Orientation Field Estimation. We want to remark some important aspects of these methods:

**Table 3.** Existing Model-Based Methods for Orientation Field Estimation.

| Method | Year |
|---|---|
| Sherlock and Monro [3] | 1993 |
| Vizcaya and Gerhardt [4] | 1996 |
| Zhou and Gu [27] | 2004 |
| Gu et. al [5] | 2004 |
| Chikkerur et al. [19] | 2007 |
| Wang et al. [6] | 2007 |
| Huckemann et al. [28] | 2008 |
| Gottschlich et al. [7] | 2009 |
| Tao et al. [29] | 2010 |
| Hou et al. [30] | 2012 |
| Feng et al. [8] | 2013 |

1. Gradient-based methods are more accurate than model-based methods only for high quality fingerprint images.
2. Several Experiments performed by Wang et al. [6] show that model-based methods are much better than gradient-based methods in the regions with strong noise or near the singular points due to the global approximation.
3. One common limitation for almost all model-based methods is the prior knowledge of singular points. It is nontrivial to retrieve a singular point from noisy fingerprint images in the first place and this matter attends against the reliability of the method. The FOMFE method does not require that and it is capable to summarize singular points areas accurately [6].
4. Another drawback in model-based methods is the need to use parameters [28]. The definition of those parameters can be a very difficult task. It can provoke decreasing in the reliability of the method.
5. Zhou and Gu explained how zero-pole model [3] and piecewise linear model [4] lack of accuracy far from singular points.
6. The prior knowledge about orientation field is unused. Therefore, a lot of valuable information obtained from the systems of recognition and verification is wasted.

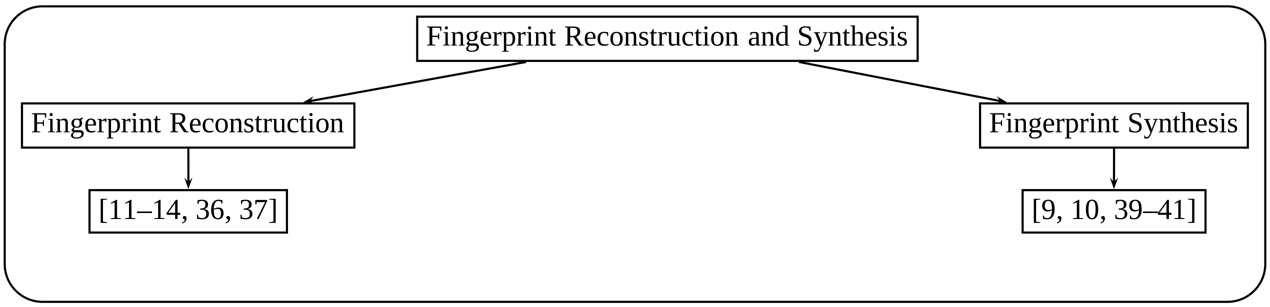
## 3 Fingerprint Reconstruction and Synthesis

Since minutiae template has become in a compact representation of a fingerprint, it has been assumed that it is not possible to reconstruct the original fingerprint from a minutiae template. The template, by definition, is a compact description of the biometric sample, it is not expected to reveal significant information about the original data. Therefore, template-generation algorithms are typically assumed to be one-way algorithms. Recently, however, this belief has been challenged by some researchers [11–14, 36, 37] who were successful in reconstructing a fingerprint image from the given minutiae template. The Analysis of fingerprint reconstruction from minutiae template is very important to determine if it is possible to fool an expert human placing a reconstructed fingerprint image in crime scenes and to perform masquerade attacks against an automatic fingerprint recognition system (for instance, injecting a reconstructed image in a communication channel or making a fake finger).

Fingerprint reconstruction can be beneficial in application like smart cards (where memory is critical) since the orientation map required for matching need not be stored explicitly but can be generated from the template. The minutiae information alone may be used for classifying fingerprints. Fingerprint reconstruction may also be used for improving the interoperability among minutiae encoders and matchers from different vendors, which was identified as a problem in the NIST MINEX testing [38].

Fingerprint reconstruction from minutia templates is very similar to fingerprint synthesis [10] except that the goals and the inputs of the two techniques are different. The goal of fingerprint reconstruction is to obtain an artificial fingerprint that resembles the original fingerprint as much as possible, while the goal of fingerprint synthesis is to generate artificial fingerprints that are as realistic as possible or regenerate a real fingerprint.

Fingerprint Reconstruction and Synthesis Taxonomy




### 3.1 Fingerprint Synthesis

Novikov and Glushenko [9] presented one model for fingerprints directional image simulation and two basic models for generating virtual ridges structure and minutiae. Their approach is based on general physical principles (minimum of energy and minimum of entropy). This model offered new opportunities for images regeneration in bad quality zones. The figure 8 shows an example of Novikov's fingerprint synthesis.

Araque et al. [39] proposed a method for synthesis of fingerprint images. Global features are condensed in a linear model whose parameters are generated according to the statistical distribution of natural fingerprint patterns. Local features have been synthesized applying recursively a simple finite state filter. Global features in fingerprints describe the ridge orientation pattern everywhere. Its synthesis is achieved through a model condensing the orientation patterns into a small set of numbers. In principle, the orientation pattern is a continuous function of the position, a continuous bidimensional variable; later it is sampled at fixed intervals vertically and horizontally resulting in a matrix whose approximation becomes the aim of the synthesis process. This matrix may be deemed as the sampling of a complex function in the complex plane. This model is based on the linear models proposed by Vizcaya [4]. Vizcaya's models generate a base set using the position of the singularities on the fingerprint. This base set is linearly combined so as the deviation of the resulting superposition from the original matrix is minimized. So, the information required to construct a reasonable approximation of the matrix is the position of the singularities together with the coefficients weighing each of the base elements.

Cappelli et al. [10] designed SFINGE, an algorithm for fingerprint synthesis. This method is very useful for creating large artificial fingerprints databases. The basic idea behind SFINGE is quite simple (see Figure 9): A fingerprint shape, a directional map, and a density map, generated independently from one another, are the inputs of a ridge-generation process; the resulting binary ridge pattern is then rendered by adding fingerprint-specific noise. In order to generate more impressions of the same finger, a more complicated schema has to be introduced: a master fingerprint (i.e., a ridge patter, which represents the unique and immutable characteristics of a "synthetic finger") must first be generated; then several synthetic fingerprints can be derived from the master fingerprint, by explicitly tuning displacement, rotation, distor-


**Fig. 8.** Regeneration of real fingerprints by adaptive resonance method. (a) Initial image distorted by white noise and $25\%$ of data loss. (c) $50\%$ data loss. (b) and (d) Restored images. Taken from [9].

tion, skin condition, and noise. Figure shows the complete generation process: Steps 1-4 create a master fingerprint; steps 5-10 are performed for each fingerprint impression derived from the master fingerprint.

Creating a master fingerprint involves the following steps:

1. Fingerprint shape generation.
2. Directional map generation.
3. Density map generation.
4. Ridge pattern generation.

Step 1 defines the external silhouette of the fingerprint. Depending on the finger size, position, and pressure on the acquisition sensor, the acquired fingerprint images can have different sizes and external shapes. The visual examination of several fingerprint images suggests that a simple model, based on four elliptical areas and a rectangle and control1ed by five parameters, can handle most of the variations present in real fingerprint shapes.

Step 2, starting from the positions of cores and deltas, exploits a mathematical flow model to generate a consistent directional map. The orientation model proposed by Sherlock and Monro [3] allows a consistent directional map to be calculated from the position of cores and deltas only. In this model the image is located in the complex plane and the local ridge orientation is the phase of the square root of a complex rational function whose singularities (poles and zeroes) are located at the same place as the fingerprint macro-singularities (cores and deltas).

Step 3 creates a density map on the basis of some heuristic criteria inferred by the visual inspection of several real fingerprints. This inspection leads they to immediately discard the possibility of generating the density map in a completely random way. In fact, they noted that usually in the region above the

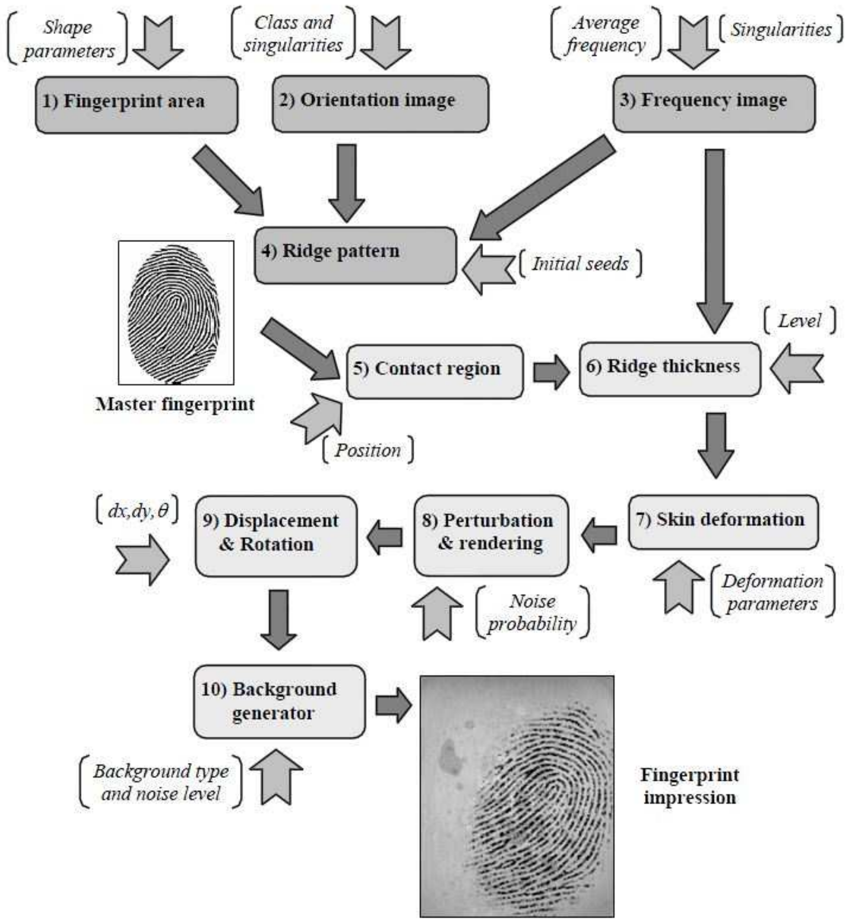



**Fig. 9.** A functional schema of the synthetic fingerprint generation according to SFINGE: Each rounded box represents a generation step; the main input parameters that control each step are reported between brackets. Steps 1-4 create a master fingerprint; steps 5-9 derive from the master fingerprint a fingerprint impression. Taken from [10].

northernmost core and in the region below the southernmost delta, the ridge line density is lower than in the rest of the fingerprint. The density map generation is performed as follows:

1. Randomly select a feasible overall background density.
2. Slightly increase the density in the above-described regions according to the singularity locations.
3. Randomly perturb the density map and perform a local smoothing reports some examples of density maps generated by this approach.

In step 4, the ridge line pattern and the minutiae are created through a space-variant iterative filtering; the output is a near-binary fingerprint image. Given a directional map and a density map as input, a deterministic generation of a ridge line pattern, including consistent minutiae, is not an easy task. The method SFINGE works iteratively enhancing an initial image (containing one or more single black pixels) through Gabor-like filters [42] adjusted according to the local ridge orientation and density, a consistent and very realistic ridge line pattern gradually appears; in particular, fingerprint minutiae of different types (endings, bifurcations, islands, etc.) are automatically generated at random positions.

One main limitation of SFINGE is that minutiae can not be controlled [12]. As a result, SFINGE may generate problematic fingerprints that contain too few minutiae or very long ridges. It is well known that the distribution of minutiae in fingerprints is not random and fingerprints of different pattern types have different minutiae distributions [37]. The minutiae distribution of fingerprints generated by SFINGE may not conform to such distributions since these minutiae are automatically generated during the image filtering process.

Zhao et al. [41] proposed a novel fingerprint image synthesis algorithm. They were looking for improve the most important shortcoming in SFINGE algorithm: the problem of minutiae. The authors explained how it is possible to get a synthetic fingerprint where minutiae to follow a more natural distribution.

The algorithm consists in four steps:

1. Sampling fingerprint features from statistical models.
2. Generating a master fingerprint.
3. Generating multiple impressions from the master fingerprint.
4. Rendering fingerprint images.

This method samples features from their statistical distribution models. Different types of fingerprint features are essentially dependent on each other. For example, orientation field is partially determined by singular points [3]. Minutiae density tends to be higher in regions around singular points than in regions far from singular points. Minutia directions are determined by their types and the ridge orientations at their locations. Therefore, given a fingerprint type to be synthesized, they sequentially sample its features from statistical models, i.e., first singular points, followed by orientation field, and finally, minutiae.

In step 2 the authors used a method proposed by Feng et al. [14] to reconstruct a master fingerprint from a set of sampled features, because it generates a relatively small number of spurious and missing minutiae in the synthetic fingerprints.

In step 3 multiple impressions from the same master fingerprint (i.e., genuine pairs of fingerprint images) are generated by distorting the master fingerprint. Nonlinear plastic distortion [43] is applied followed by global rigid transformation (i.e., rotation and translation).

The objective of step 4 is to add more realistic conditions to the synthetic fingerprint. Therefore, they render the impressions by simulating finger dryness and adding noise.

**Table 4.** Existing Fingerprint Synthesis Methods.

| Method | Input | Model | year |
|---|---|---|---|
| Novikov and Glushchenko [9] | ridge orientation | iterative filtering | 1997 |
| Araque et al. [39] | ridge orientation and frequency | second-order orientation model; filtering using binary mask | 2002 |
| Cappelli et al. [10] | singular points, shape parameters and average frequency | orientation model of Sherlock and Monro [3] | 2003 |
| Bicz [40] | minutiae, ridge orientation and frequency | Frequency Modulation Model | 2003 |
| Zhao et al. [41] | fingerprint type, image size statistical feature models | orientation model of Sherlock and Monro [3] | 2012 |

## 3.2 Fingerprint Reconstruction

In this section we describe several methods for fingerprint reconstruction.

Ross et al. [11] proposed a technique for fingerprint reconstruction that use minutiae triplet information to estimate the orientation map of the parent fingerprint. The estimated orientation map is observed to be remarkably consistent with the underlying ridge flow. The algorithm for generating the orientation map has four main stages:

1. Triplet generation.
2. Orientation prediction.
3. Triplet pruning.
4. Orientation smoothing.

We have an example in figure 10.

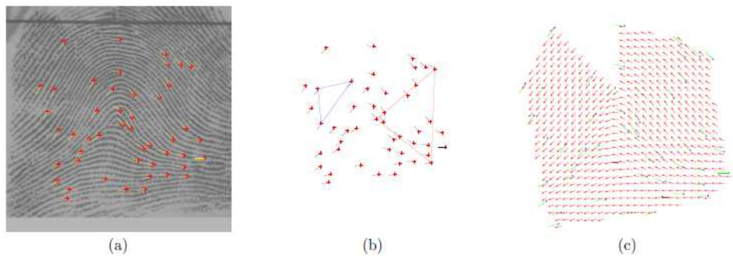



**Fig. 10.** (a) Minutiae distribution of a fingerprint. (b) Examples of a good quality triplet (blue) and a bad quality triplet (red). (c) Estimated orientation map. Taken from [11].

The proposed algorithm of fingerprint reconstruction is based in the Gabor-like filter [42]. The algorithm performance:

1. An empty fingerprint image of size 512 x 512 is divided into non-overlapping blocks.
2. Each block is associated with an orientation value O(z) estimated with the algorithm. Many blocks may not have orientation information since the estimated orientation map can be incomplete.
3. The block is next initialized with a noisy blob and is convolved with the Gabor filter whose parameters are tuned using O(z). This results in a new image, which is again subjected to the convolution procedure.
4. This process is repeated k times resulting in an image which exhibits ridge-like patterns.

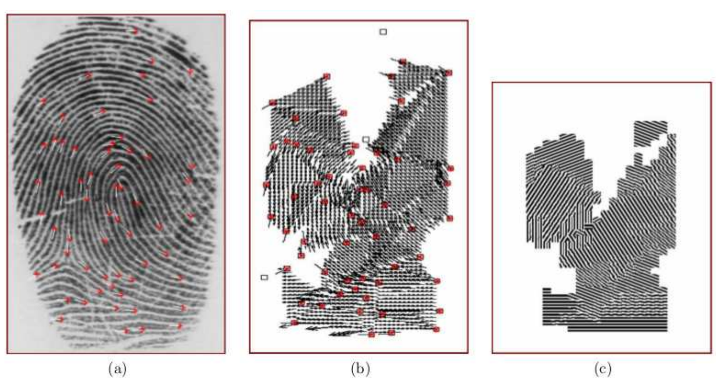



**Fig. 11.** Reconstructing fingerprints. (a) Minutiae distribution of a fingerprint image. (b) Predicted orientation map (c) Reconstructed fingerprint. Taken from [11].

Figure 11 shows an example of fingerprint reconstruction from the estimate orientation map. This technique has a common problem: the reconstruction of the image is partial.

These researchers proposed another method for Reconstructing Fingerprints in [37]. They do the reconstruction from minutiae template. They obtain the orientation field information, the class or type information and the friction ridge structure for parent fingerprint. The orientation estimation algorithm determines the direction of local ridges using the evidence of minutiae triplets [11]. The estimated orientation field and minutiae distribution are used to predict the class of the fingerprint. Finally, the ridge structure of the parent fingerprint is generated using streamlines that are based on the estimated orientation field (see figure 12) and Line Integral Convolution (LIC) [44], this is basically a texture synthesis technique that is used to visualize 2D data. The algorithm is as follows:

1. Estimating orientation map [11].
2. Constructing streamlines using orientation map. The first action for this step is seed point selection. These points are the origin of streamlines in the orientation map. After the streamlines are constructed using linear interpolation scheme.
3. Generating ridge structure using LIC. Given a streamline the LIC technique involves calculating the intensity of all pixels constituting the streamline. It locally blurs an uncorrelated input texture image, such as white noise, along the path of the streamlines to impart a dense visualization of the flow field.
4. Enhancing the ridge map. In order to increase the ridge width, they use a lowpass filter to smooth the texture image generated using LIC and then perform histogram equalization of the ridge structure for contrast enhancement.

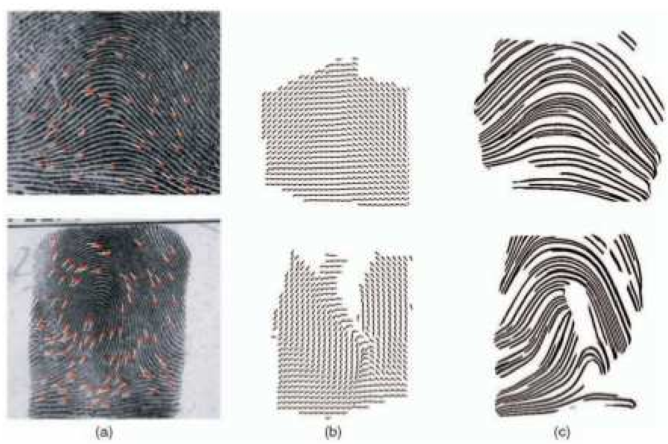



**Fig. 12.** Reconstructing the ridge structure. (a) Original fingerprint and its minutiae plot. (b) Estimated orientation map. (c) Enhanced ridge structure after application of the Verifinger software. Taken from [12].

Cappelli et al. [13] presented a novel approach to reconstruct fingerprint images from standard templates and they investigate to what extent the reconstructed images are similar to the original ones. Reconstruction approach is based on a sequence of steps that receive the minutia template and attempt to estimate various aspects of the original unknown fingerprint (figure 13): the fingerprint area, the orientation image and the ridge pattern.

They reconstruct the fingerprint area using a simple mathematical model introduced in [10]. The orientation model adopted in that work was originally proposed in [4] and extended in [10] to enable the generation of synthetic orientation images. Given the minutiae set, the estimated orientation image and the frequency, the ridge pattern reconstruction involves the followings steps:

1. Minutiae prototype positioning.

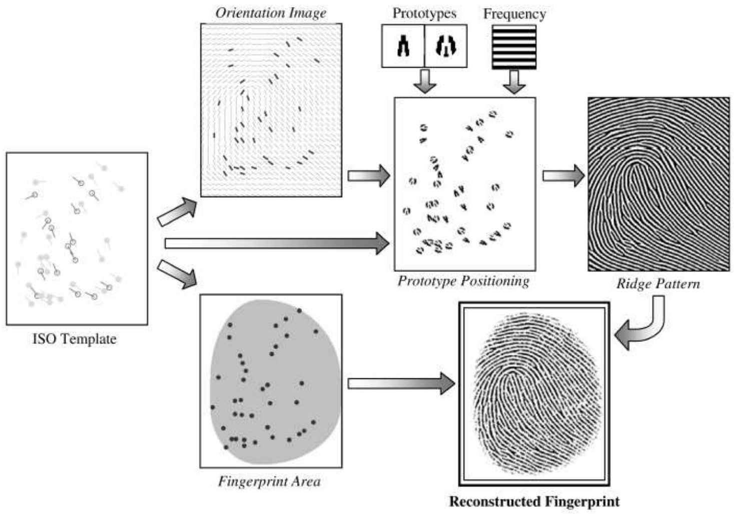



**Fig. 13.** Algorithm of the reconstruction approach. Taken from [13].

2. Iterative pattern growing.

Step 1 is completed using the information of minutia and frequency.

Step 2 iteratively grows the minutia prototypes by applying at each pixel a Gabor filter adjust according to the frequency and the local orientation.

Feng and Jain [12] proposed an algorithm to reconstruct fingerprint from minutiae template minimizing the problem of reconstruct a partial fingerprint and the another problem: many spurious minutiae not included in the original minutiae template are generated in the reconstructed template. This algorithm receives the minutiae template:

$$\{X_n, Y_n, \partial_n\}, \tag{15}$$

where $(X_n, Y_n)$ and $\partial_n$ are location and direction of minutiae respectivatly.

The following three steps are performed to obtain the reconstruct image (see figure 14):

1. Orientation field reconstruction.
2. Continuous phase reconstruction.
3. Combination of the spiral phase and the continuous phase.

The algorithm was evaluated matching original fingerprint against reconstructed fingerprint and match reconstructed fingerprint against different impressions of the original fingerprint.

Later in [14], the same authors proposed another algorithm for fingerprint recognition. In this algorithm a fingerprint image is represented as a phase image which consists of the continuous phase and the spiral phase. This algorithm minimize the problem of spurious minutiae and the reconstruction the partial image like [12].

The steps of the algorithms are followings (see figure 15):

1. Orientation field reconstruction.
2. Estimation of gradient of continuous phase.
3. Continuous phase reconstruction.
4. Combination of the spiral phase and the continuous phase.

The figure 16 shows a reconstructed fingerprint by Feng and Jain's method.

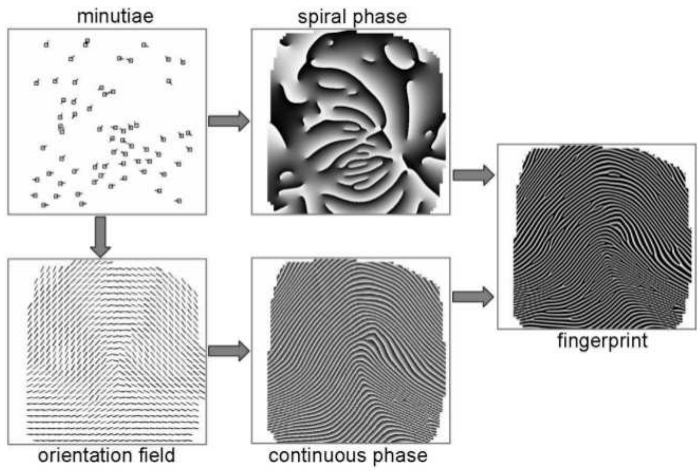



**Fig. 14.** Flow chart of the proposed fingerprint reconstruction algorithm. Taken from [12].

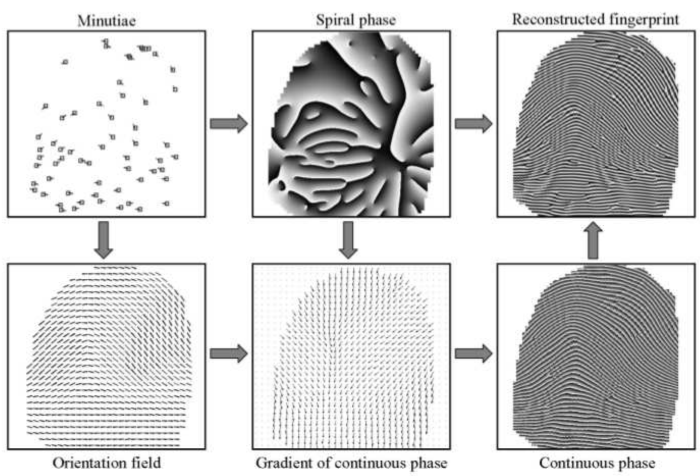



**Fig. 15.** Flow chart of the proposed fingerprint reconstruction algorithm. Taken from [14].

**Table 5.** Existing Fingerprint Reconstruction Methods.

| Method | Input | Model | year |
|---|---|---|---|
| Hill [36] | singular points, minutiae | orientation model of Sherlock and Monro [3]; line drawing | 2001 |
| Ross et al. [11] | minutiae | minutiae triplets; gabor filter | 2005 |
| Ross et al. [37] | minutiae | minutiae triplets; stream lines; Line Integral Convolution | 2007 |
| Cappelli et al. [13] | minutiae | orientation model of Vizcaya [4]; Gabor filtering | 2007 |
| Feng and Jain [12] | minutiae | Frequency Modulation Model | 2009 |
| Feng and Jain [14] | minutiae | Frequency Modulation Model | 2011 |

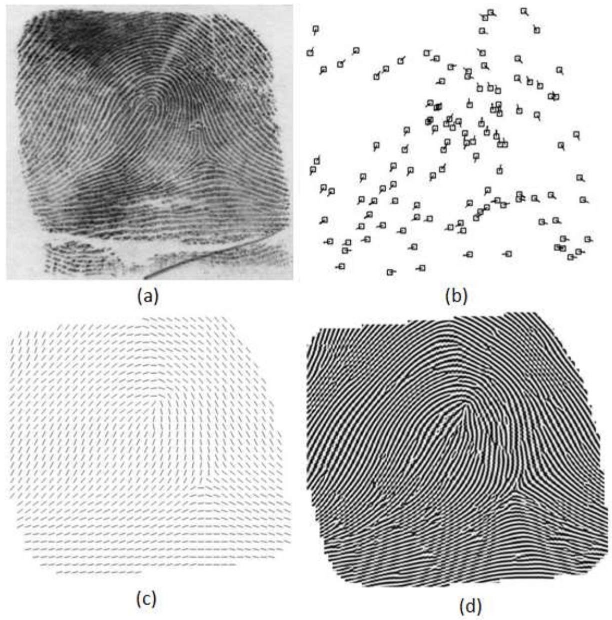


**Fig. 16.** An example of fingerprint reconstruction by Feng and Jain [14]. (a) Original fingerprint; (b) minutiae template; (c) orientation field estimation; (d) reconstructed fingerprint.

### 3.3 Main Drawbacks Detected

The issue of fingerprint reconstruction has many important applications. Reconstructing ridges structure you can enhance your fingerprint and improve the matching because recently some techniques of fingerprint matching are based in ridge structure. When an expert analyse a latent he can detect minutiae and makes the minutia template. This template may be used for reconstructing latent and improving very much the efficacy of matching algorithms. The analysis of fingerprint reconstruction from minutiae templates show that it is very improbable to fool a human expert but it is probable to perform masquerade attacks against an automatic fingerprint recognition system.

In this research we detect many drawbacks in the fingerprint reconstruction from minutiae template. They are following:

1. Usually it is not possible reconstruct a whole fingerprint image and the reconstruction is only partial: this problem was tried by [12].
2. When the fingerprint is reconstructed many spurious minutiae could be included: this problem was tried by [12] and [14].
3. In zones with high curvature the quality of the reconstructed images is lower than others zones. These zones are important because almost singular points are located here.
4. These algorithm need a lot of minutiae in the template to increase the quality of the reconstructed image and for doing the reconstruction more complete. That is a drawback in latent fingerprint and in application where memory is critical to storage minutia information.

## 4 Conclusions

As a conclusion of this review we can remark the importance of the orientation field estimation for many problem within fingerprint identification and recognition such as enhancement, classification, singular point detection, reconstruction and matching.

We studied the use of several mathematical models for orientation field reconstruction. Their performance for low quality image are very high. The analysis showed how these models are better than gradient based methods in those cases where the quality of the image is low, due to their global characteristics.

The fingerprint synthesis is very useful for regenerating images where the ridges are broken or lost and to build large databases of artificial fingerprint.

The fingerprint reconstruction for standard minutiae templates is very useful too in some application where the memory is critic and for improving the interoperability among minutiae encoders and matchers from different vendors. Their study it is important for the prevention of the masquerade attacks against identification systems. We identified some drawbacks in the current methods in the state of the art.

We propose as future work the development of a new method for fingerprint reconstruction using others points and also minutiae template. The new points could be selected by an expert human or by a system. Those points should offer additional information with the objective to obtain a more complete and reliable reconstructed fingerprint. We are thinking in the use of singular points, high curvature points and others that can serve as new sources of information. That new approach would introduce changes in the representation of fingerprint in the minutiae template but it could be beneficial for fingerprint reconstruction.

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